Comparative Study of Different Cost Functions between Neighbors for Optimizing Energy Dissipation in WSN

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Abstract—Life time of battery is one of the major concerns in Wireless Sensor Networks (WSNs). Traditional network algorithm chooses the data transmission path based on the distance between sender and receiver. For example, in Directed Diffusion (DD), the nodes closer to the sink node are more active so that the continuous data flow of the network is maintained. However, in such scenario, there is an energy hole problem because the nodes closer to the sink reduce their energy since they are more active. But if the residual energy and shortest hop counts can be taken together, then the problem can be overcome. We have analyzed different transmission cost functions with respect to Approximated Uniform Energy Dissipation Directed Diffusion Algorithm (AUEDDD). The final goal of the paper is to find out the suitable cost function to resist the early death of the first node of a network. We introduce dynamic priority variable, which is a tradeoff between total energy consumption and uniform energy dissipation.


I. INTRODUCTION

Wireless Sensor Networks (WSNs) can be used for various purposes, such as disaster management, environmental monitoring, patient monitoring, among others [1]. Different routing algorithms are used according to requirements. Each routing algorithm has its own merits and demerits and is useful only for certain applications. Energy is the most valuable resource in wireless sensor nodes[2]. This is mainly because, in most of the cases, sensors are deployed in remote places with almost zero probability of regular maintenance. Therefore renewal/changing of the source of energy is next to impossible. Hence, lower energy consumption by nodes implies longer lifespan of a WSN. There is a need for an efficient routing algorithm so that the rate of energy dissipation can be reduced, thereby enhancing effective lifetime of a sensor node. Here, we consider the effective lifetime of a WSN to be the time till the first node in the network runs out of energy.

This paper explores alternative techniques to avoid problems arising out of energy consumption and its effect on WSN lifetime. The objective is to strike a balance between minimizing energy consumption for data transmission and maximizing the network lifetime by using a path selection algorithm, at the same time. The energy dissipation during transmission should be uniformly distributed among the nodes on different routes along the path from source to destination. Efficient path selection can be applied here where a path is established before packet transmission has begun. This paper proposes an optimum cost function which can maximize the effective life time of WSN. With similar parameters, we can construct different cost functions and find out which cost function will provide best result with respect to others. This paper further analyzes different cost functions for finding out the optimum one by doing theoretical analysis and simulating different types of cost functions on top of AUEDDD algorithm [3]. We use dynamic priority variable of cost function for ensuring the tradeoff between total energy dissipation and uniform energy dissipation. Initially, while all the nodes contain sufficient energy then shorter path can be selected for message transmission by allowing greater energy dissipation per node. Gradually nodes in the network lose energy. To avoid creation of energy hole or early death of first node, uniform energy dissipation takes over. Uniform energy dissipation may be defined to be the consumption of energy at the same rate by participant nodes in a network. Hence, uniform energy dissipation means the objective to achieve energy dissipation in the same order for all nodes in the network. Lifetime enhancement may be achieved in a network by focusing (i) on minimizing total energy consumption (which is equivalent to maximizing total energy conservation) in the network, and/or (ii) on the uniformity of energy consumption by each node.

The organization of the rest of the paper is as follows.
Section II discusses related existing work on optimization of energy dissipation in WSNs. Section III discusses the Approximate Uniform Energy Dissipation Directed Diffusion algorithm[3]. Different types of cost functions are discussed in Section IV, that are theoretically compared in Section V. Tradeoff between total energy dissipation and uniform energy dissipation is discussed in Section VI. Section VII compares different priority variables and proposes dynamic priority variables for getting better results. The performance results are given in Section VIII. Section IX provides a discussion while Section X concludes the paper.

II. RELATED WORK OF ENERGY EFFICIENCY IN WSN

To prolong the life time of WSNs, researchers are working in different aspects of these systems. In [4] authors proposed a circular non-uniform node distribution. The nodes relatively closer to the sink node will dissipate energy much earlier than the nodes relatively away from sink node due to over activity. They divided the network area into several co-centric corona by placing the sink node at the center of these corona. Non-uniform node distribution is used to achieve near balanced energy depletion. The work in [5] addresses optimum node density policy to increase the life time of WSNs. The authors showed that if any node resides within a certain distance i.e.

\[
\min \left\{ t_s, \left( \frac{2c}{\alpha - 2} \right)^{\frac{1}{\alpha}} \right\}
\]

where \( t_s \) is the transmission range, \( \alpha \) is the path loss coefficient and \( c \) is a constant. The message needs to be transmitted directly to the sink node. They also proposed new corona based deployment strategy to prolong the life time of network. Li et al. [6] studied a generic framework for energy constrained distributed estimation in WSNs. They optimized energy consumption with respect to number of node alive in the network. Different works proposed various energy efficient clustering techniques for optimizing energy dissipation in case of proactive routing algorithms. Different researchers provide different types of routing algorithm having different advantages like LEACH[7], HEED[8], PEGASIS[9]. All these routing algorithms are as are proactive routing algorithms. Applying energy efficient MAC protocols can increase the life time of the network. Maitra et al [10] compared different MAC protocols using simulation analysis. MAC protocols save energy by saving duty cycle of the WSNs. Energy can be optimized by using the efficient data gathering and data fusion algorithm[11][12][13] in order to reduce the redundancy in case of the message transmission. By using optimum path selection algorithms, the life time of WSNs can also be increased. In [14] sensors in the network have a choice of different coding schemes to achieve varying levels of compression. At first a routing strategy is selected and then an optimal combination of data representation algorithms is chosen at each node. The authors showed that overall energy consumption can be significantly reduced by optimizing the coding algorithm selection, with respect to the case when data is simply quantized and forwarded to the central node. The work in [15] uses the principle of opportunistic routing theory. The distance of a sensor node from its sink and the residual energy of both are considered while making the multi hop relay decision to optimize network energy efficiency. The authors designed an Energy Saving via Opportunistic Routing (ENS_OR) algorithm that ensures minimum power cost during data relay. The algorithm also protects the nodes with relatively low residual energy. The authors propose a Centralized Energy Efficient Distance (CEED) based routing protocol in [16]. The protocol is aimed at even distribution of energy dissipation among all sensor nodes. Based on LEACH’s energy dissipation model, optimum number of cluster heads are calculated. The authors proposed a distributed cluster head selection algorithm based on dissipated energy of a node and its distance to Base Station. The authors also extended the proposed protocol by multi hop routing scheme to reduce energy dissipated by nodes located far away from base station. The authors in [17] are concerned with maintaining the topology of the WSN. It is well known that use of Connected Dominating Sets (CDS) is promising in topology control. The work addresses the problem of constructing energy efficient CDS in WSNs while improving network reliability. The authors visualize the multi-objective optimization that simultaneously maximizes two contradictory parameters: reliability and energy efficiency. The works discussed so far mainly concentrate on node deployment and optimum path selection algorithms in addition to efficient data gathering and fusion.

In [3], the authors proposed an algorithm called AUEDDD, where some cost functions for message transmission and a probability function for next node selection have been discussed. They have also discussed some priority variable (\( \alpha, \beta \)) which prioritizes different parameters of the function. In this paper we have proposed the use of dynamic priority variable in the cost functions as tradeoff between life time of the network and total energy savings in the network. Section VI provides theoretical proof that dynamic priority variables in the cost functions are much more advantageous than static variables. In Section VII we provided the graphs (Fig. 8 to Fig. 13) that show that the use of dynamic priority variable is better approach than the use of static variable. In this paper, next node selection in forward gradient is done by keeping remaining energy of a node, in mind. We have analyzed the transmission cost functions, probability function and redefined those priority variables dynamically. In this paper we have assumed that initially all nodes have same amount of energy. The comparative study of different cost function is also valid if the initial energy of nodes is not equal. Different cost functions follow probabilistic approach to select next node for sending message. Because after sending subsequent number of messages the nodes of the network contain heterogeneous energy amount of energy. The theoretical analysis and simulation results proved different types of cost functions are able to handle that situation. In [18] authors consider that nodes can harvest energy at their idle time; for that reason the
nodes can have evenly distributed energy. Even then also, the probabilistic cost functions discussed in this paper can work efficiently.

III. APPROXIMATED UNIFORM ENERGY DISSIPATION DIRECTED DIFFUSION ALGORITHM (AUEDDD)

AUEDDD algorithm is approximated routing algorithm, which performs tradeoff between total energy consumption and per node energy consumption. By minimizing total energy consumption, it prolongs average life time of all WSN nodes. Also by minimizing the standard deviation of per node energy dissipation it prolongs the first node death of network. Here Table I lists the symbols used in this paper.

<table>
<thead>
<tr>
<th>Table I</th>
<th>SYMBOLS USED IN THE PAPER</th>
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<tr>
<td>$C_{ji}$</td>
<td>Transmission cost from node $j$ to node $i$</td>
</tr>
<tr>
<td>$C_{ji}^I$</td>
<td>Type I cost function</td>
</tr>
<tr>
<td>$p_{ji}$</td>
<td>Probability of choosing next node for transmission from node $j$ to node $i$</td>
</tr>
<tr>
<td>$p_{ji}^I$</td>
<td>Type I cost probability function</td>
</tr>
<tr>
<td>$e_{ji}$</td>
<td>Energy required to transmit a message from node $j$ to node $i$</td>
</tr>
<tr>
<td>$FGT_j$</td>
<td>Forward gradient table.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Priority variables</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Priority variables</td>
</tr>
<tr>
<td>$e_{rem}^x$</td>
<td>Remaining energy of node $x$.</td>
</tr>
</tbody>
</table>

According to [3], the transmission cost function from node $j$ to node $i$ is $C_{ji}$.

$$C_{ji} = \{e_{ji}\}^\alpha \{e_{rem}^i\}^\beta$$ (1)

Here $e_{ji}$ is the minimum energy dissipation, $e_{rem}^i$ is the remaining energy of node $i$, and $\alpha$, $\beta$ are priority variables. The probability of choosing a node from different neighbor nodes is [1]:

$$P_{ji} = \{1/C_{ji}\} / \{ \sum_{k \in FGT_j} 1/C_{ji} \}$$ (2)

where $C_{ji}$ is the cost of message transmission between node $j$ and node $i$ and $FGT_j$ is the Forward Gradient Table of node $j$. The Forward Gradient Table [3] contains the node ID of the next node to which a packet is to be sent, remaining energy of the next node and distance between current node and next node. Similarly, the Reverse Gradient Table contains the ID of the previous node from which a packet has been received, remaining energy of previous node and distance between current node and previous node. Both tables are established at the time of Gradient setting. In [3], the cost function (equation (1)) and path selection probability function (equation (2)) are adapted. It has been shown in [3] that by considering different value for $\alpha$ and $\beta$ the life time of WSNs and average of per node energy of message transmission changes differently. By increasing the value of $\beta$ per node, energy saving and uniform energy saving get higher priority. Whereas by reducing the value of $\beta$ with respect to the value of $\alpha$, total energy dissipation gets higher priority.

We denote the probabilities of choosing nodes $x$ and $y$ as next nodes for sending data from node $j$ as $p_{jx}$ and $p_{jy}$ respectively. Here $e_{rem}^x$ and $e_{rem}^y$ denote remaining energy of nodes $x$ and the probability of choosing that node is $p_{jx}$ and $y$ (the probability of choosing that node is $p_{jy}$), respectively.

As discussed above, both nodes $x$ and $y$ are members of the Forward Gradient Table of $j$ ($FGT_j$). Therefore, we can say:

$$P_{jx} - P_{jy} \leq \varepsilon$$ (3)

$$(e_{rem}^x)^\beta - (e_{rem}^y)^\beta \leq \varepsilon \sum_{k \in FGT_j} (e_{rem}^x)^\beta$$

Hence, $\{e_{avg}\}^\beta = \sum_{x=1}^{r} (e_{rem}^x)^\beta / \varepsilon$ where $e_{avg}$ is the power average of $\{(e_{rem}^x)^\beta\}$ where $1 \leq x \leq r$

$$(e_{rem}^x)^\beta - (e_{rem}^y)^\beta \leq \varepsilon (e_{avg})^\beta$$ (4)

When $e_{rem}^y$ $\approx$ 0 and $(e_{rem}^x)^\beta - (e_{rem}^y)^\beta \leq \varepsilon (e_{rem_{avg}})^\beta$ then we can say

$$(e_{rem}^x)^\beta \leq \varepsilon (e_{avg})^\beta$$ (5)

It is obvious that as the energy of individual neighboring nodes decreases, the average remaining energy also decreases. Since the value of $e_{rem}^y$ is almost zero, the value of $e_{avg}$ will also tend to zero since the nodes has been chosen based on higher remaining energy criterion. Also, if the value of $\varepsilon$ is
very small with respect to other parameters of equation (5), then the product of $\varepsilon$ and $r(e_{av})^\beta$ will be negligible. Hence, it can be said that the value of $e_{rem}^\alpha$ is small when the value of $e_{rem}^\alpha = 0$. Therefore, if the value of $\varepsilon$ increases then the total amount of energy conservation will increase with minor violation to uniform energy dissipation rule. Therefore there will be very little chance that one node has huge energy whereas another is dying. Simulation results support our theoretical claim.

IV. COST FUNCTIONS INVOLVED IN MESSAGE TRANSMISSION

According to AUEDDD, the cost function of transmission is:

$$C_{ji} = (e_{ji})^\alpha / (e_{rem})^\beta$$

(6)

This cost function contains four variables $(e_{ji}, e_{rem}, \alpha, \beta)$. Equation (6) shows that transmission cost will increase with increase in the value of $e_{ji}$, and transmission cost will decrease with increasing value of $e_{rem}$. Keeping in mind the above fact, we propose following four possible types of cost functions; namely Type I, Type II, Type III, and Type IV functions. With respect to basic mathematical operation only these four type of cost functions can be constructed with four parameters $(e_{ji}, e_{rem}, \alpha, \beta)$ and the previously mentioned condition.

A. Type I cost function

According to [3], we calculate the cost of message transmission between any two nodes (from node $j$ to node $i$) denoted by $C_{ji}$. Here the variables $\alpha$ and $\beta$ are used as exponential factors. The notation $C_{ji}^I$ denotes Type I cost function and is defined as follows:

$$C_{ji}^I = (e_{ji})^\alpha / (e_{rem})^\beta$$

(7)

B. Type II cost function

In case of Type II cost function, the priority variables $\alpha$ and $\beta$ are used as multipliers of $e_{ji}$ and $e_{rem}$ respectively and product of $\alpha e_{ji}$ and $1/\beta e_{rem}$ is denoted as the Type II cost function. The notation for the Type II cost function is $C_{ji}^{II}$. $C_{ji}^{II}$ is defined as:

$$C_{ji}^{II} = \alpha e_{ji} / (\beta e_{rem})$$

(8)

C. Type III cost function

Type III cost function is obtained when $\beta e_{rem}$ is subtracted from $\alpha e_{ji}$ as follows:

$$C_{ji}^{III} = \alpha e_{ji} - \beta e_{rem}$$

(9)

D. Type IV cost function

Using priority variables $\alpha$ and $\beta$ as exponents of $e_{rem}$ and $e_{ji}$ instead of multipliers as in Type II, we get the Type IV cost function. The expression for Type IV cost function is given below:

$$C_{ji}^{IV} = (e_{ji})^\alpha - (e_{rem})^\beta$$

(10)

V. ANALYSIS OF DIFFERENT TYPES OF COST FUNCTIONS

In the previous section, different types of cost functions are defined. These cost functions have their own merits and demerits. In this section, we have analyzed and compared different cost functions with respect to energy conservation and computational complexity.

A. Comparative study of Type I and Type II cost functions

The probability of choosing node $i$ as the next node in case of Type I cost function is:

$$P_{ji}^I = \frac{(e_{rem})^\alpha / (e_{ji})^\beta}{\sum_{l \in FIG_j} (e_{rem})^\alpha / (e_{ji})^\beta}$$

(11)

Similarly, the expression for the probability of choosing node $i$ as the next node in case of Type II cost function is:

$$P_{ji}^{II} = \frac{(e_{rem})^\alpha / (e_{ji})^\beta}{\sum_{l \in FIG_j} (e_{rem})^\alpha / (e_{ji})^\beta}$$

(12)

Since probability calculation in case of Type II cost function is independent of priority variables, therefore, it can be said that Type II cost function is basically Type I cost function having the value of all priority variables to be one. We use the priority variables to emphasize any parameter with respect to other parameters in the function. The Type II probability function is basically special type of Type I probability function where the value of $\alpha$ and $\beta$ is equal to one.

B. Comparative study of Type I and Type III cost functions

Since exponentiation operator is used in Type I function, computational complexity in Type I function is higher than Type III function. In Type III function, the expression for the probability of choosing node $i$ as the next node is:

$$P_{ji}^{III} = \frac{\alpha (kd_{ji}^n - \beta e_{rem})}{\alpha \sum_{l \in FIG_j} (kd_{ji}^n) - \beta \sum_{l \in FIG_j} (e_{rem})}$$

(13)

Where $k$ is the constant (permittivity of the medium). If we apply Type III cost function in case of equation (3) then we get:

$$\frac{\alpha (kd_{ji}^n) - \beta e_{rem}}{\alpha \sum_{l \in FIG_j} (kd_{ji}^n) - \beta \sum_{l \in FIG_j} (e_{rem})}$$

(14)
When the remaining energy of node \( y \) becomes zero then the expression (14) will be:

\[
\beta (e^x_{\text{rem}}) - \alpha (kd^n_{jx}) + \alpha (kd^n_{jp}) \\
\leq \epsilon \left( \sum_{l \in \text{FTG}_j} (e^x_{\text{rem}})^{\beta} - \alpha \sum_{l \in \text{FTG}_j} (kd^n_{jl})^{\beta} \right)
\]  
(16)

Since the value of \( \alpha (kd^n_{jp}) \) is low with respect to the other parameters, we can ignore that factor in equation (16):

\[
e^x_{\text{rem}} \leq \epsilon \left( \sum_{l \in \text{FTG}_j} (e^i_{\text{rem}})^{\beta} - \alpha \sum_{l \in \text{FTG}_j} (kd^n_{jl})^{\beta} \right)
\]
(17)

\[
e^x_{\text{rem}} \leq r \epsilon \left( e_{\text{avg}} - \frac{\alpha}{\beta} kd^n_{\text{avg}} \right)
\]
(18)

When \( e^x_{\text{rem}} \) becomes zero then \( e_{\text{avg}} = \frac{\alpha}{\beta} kd^n_{\text{avg}} \)

Since \( r \epsilon \neq 0 \). If \( r > \alpha \), we can ignore the amount of energy \( \frac{\alpha}{\beta} kd^n_{\text{avg}} \) for message sending. Therefore, when \( r > \alpha \) the expression for \( e^x_{\text{rem}} \) in Type III function will be:

\[
e^x_{\text{rem}} \leq r \epsilon (e_{\text{avg}})_{\text{III}}
\]
(19)

When the remaining energy of node \( y \) becomes zero then the expression for the \( e^x_{\text{rem}} \) in case of Type I cost function is:

\[
e^x_{\text{rem}} \leq (r \epsilon)^{1/\beta} (e_{\text{avg}})_{I}
\]
(20)

If the value of \( \beta \) is greater than one, then we can say the value of \( r \epsilon \) will be greater than the value of \( (r \epsilon)^{1/\beta} \). When the value of \( \beta \) is greater than the value of \( \alpha \) then \( \beta \) will impose higher priority to the remaining energy of any node, which leads to uniform energy dissipation criterion. Therefore, it can be stated that while the value of \( e^x_{\text{rem}} \) is zero then the value of \( (e_{\text{avg}})_{I} \) will be less than the value of \( (e_{\text{avg}})_{\text{III}} \).

Therefore, we can say that the computational complexity of Type III function is lower than Type I cost function, but in case of uniform energy dissipation criteria, Type I cost function performs better than Type III cost function.

C. Comparative study of Type I and Type IV cost functions

From the expression for Type III cost function where priority variables \( \alpha \) and \( \beta \) act as respective exponent of \( e^i_{\text{rem}} \) and \( e_{ji} \), we can get Type IV function.

\[
C^x_{ji} = (e_{ji})^\alpha - (e^i_{\text{rem}})^\beta
\]
(21)

If we apply Type IV cost function in equation (3) we get:

\[
\frac{(kd^n_{ji})^\alpha - (e^i_{\text{rem}})^\beta}{\sum_{l \in \text{FTG}_j} (kd^n_{jl})^\alpha - \sum_{l \in \text{FTG}_j} (e^i_{\text{rem}})^\beta} - \frac{(kd^n_{ji})^\alpha - (e^i_{\text{rem}})^\beta}{\sum_{l \in \text{FTG}_j} (kd^n_{jl})^\alpha - \sum_{l \in \text{FTG}_j} (e^i_{\text{rem}})^\beta} \leq \epsilon
\]
(22)

When the remaining energy of node \( y \) becomes zero then the expression (22) will be:

\[
(e^x_{\text{rem}})^\beta - (kd^n_{ji})^\alpha + (kd^n_{ji})^\alpha \leq \epsilon \left( \sum_{l \in \text{FTG}_j} (e^y_{\text{rem}})^\beta - \sum_{l \in \text{FTG}_j} (kd^n_{jl})^\alpha \right)
\]
(23)

Since the value of \( k^\alpha \{ (d^n_{ji})^\alpha - (d^n_{ji})^\alpha \} \) is low with respect to \( (e^x_{\text{rem}})^\beta \), we can ignore \( k^\alpha \{ (d^n_{ji})^\alpha - (d^n_{ji})^\alpha \} \) in the equation (23). Therefore, we can write the equation (23) as:

\[
e^x_{\text{rem}} \leq r e^{1/\beta} \left( \sum_{l \in \text{FTG}_j} (e^i_{\text{rem}})^\beta - \sum_{l \in \text{FTG}_j} (kd^n_{jl})^\alpha \right)^{1/\beta}
\]
(24)

If the value of \( \beta \) is greater than one, then the value of \( r \epsilon \) will be greater than the value of \( (r \epsilon)^{1/\beta} \). When the value of \( \beta \) is greater than the value of \( \alpha \) then \( \beta \) will impose much priority to the remaining energy of any node, which leads to uniform energy dissipation criterion. While considering the remaining energy of node \( x \) and \( \beta > \alpha \) we can ignore the value of \( (kd^n_{\text{avg}})^\alpha \) with respect to \( (e_{\text{avg}})^\beta \). Therefore, the expression for the \( e^x_{\text{rem}} \) in case of Type IV function will be:

\[
e^x_{\text{rem}} \leq (r \epsilon)^{1/\beta} \left( e_{\text{avg}} \right)^\beta - (kd^n_{\text{avg}})^\alpha
\]
(25)

Hence, it can be said while the value of \( e^x_{\text{rem}} \) is equal to zero then the value of \( (e_{\text{avg}})_{IV} \) will be same as the value of \( (e_{\text{avg}})_{III} \). So, in both cases (Type I and Type IV cost functions), it can be said that when the remaining energy of node \( x \) is zero then the remaining energy of other neighbor nodes is also zero. Since computational complexity of division operation is higher than the addition operation then we can say that computational complexity of Type I function is higher than the Type IV function.

D. Comparative study of Type III and Type IV cost functions

The expression for the remaining energy of node \( x \) in case of Type III function is:

\[
e^x_{\text{rem}} \leq \epsilon \left( \sum_{l \in \text{FTG}_j} (e^i_{\text{rem}})^\beta - \alpha \sum_{l \in \text{FTG}_j} (kd^n_{jl})^\alpha \right)
\]
(26)

When the value of \( e^x_{\text{rem}} \) is equal to zero then according to Type III cost function, we can write:

\[
\sum_{l \in \text{FTG}_j} (e^i_{\text{rem}})^\beta = \alpha \sum_{l \in \text{FTG}_j} (kd^n_{jl})^\alpha
\]
(27)

Let us assume the expression of \( (kd^n_{ji})_{\text{III}} \) and \( (e^x_{\text{rem}})_{\text{III}} \) in case of Type III function to be denoted as \( A_i \) and \( B_i \) respectively.
When the value of $e^x_{rem}$ is equal to zero, then according to the Type IV cost function we can write:

$$\sum_{l \in FTG_j} (e^x_{rem})^\beta_{IV} = \sum_{l \in FTG_j} (kd^n_{ji})^\beta_{IV}$$

(28)

Since we are doing this study assuming the same network topology, the value of the expression $kd^n_{ji}$ is the same in case of any type of function. Therefore, in case of Type IV function we also can denote the expression $(kd^n_{ji})^\beta_{IV}$ as $A_l$. Let us assume that the expression for $(e^x_{rem})^\beta_{IV}$ in case of Type IV cost function is $C_l$. Therefore, the expressions for (27) and (28) will be:

$$\beta \sum_{l \in FTG_j} B_l = \alpha \sum_{l \in FTG_j} A_l$$

(29)

$$\sum_{l \in FTG_j} (C_l)^\beta = \sum_{l \in FTG_j} (A_l)^\alpha$$

(30)

Since energy dissipation is uniform over the network in both types of cost functions (Type III and Type IV), therefore it can be assumed that the standard deviation of $B_l$ and $C_l$ will be low. Let us assume $C_l = \eta_l B_l$, where the value of $\sum_{l \in FTG_j} \eta_l$ is possible minimum number. We also assume $[\alpha, \beta, A_l, B_l, C_l] > 1$. Since $\beta \sum_{l \in FTG_j} B_l = \alpha \sum_{l \in FTG_j} A_l$ and $\beta > \alpha$ then we can say:

$$\sum_{l \in FTG_j} (B_l)^\beta > \sum_{l \in FTG_j} (A_l)^\alpha$$

(31)

If we replace the value of $C_l$ with $\eta_l B_l$ then equation (30) will become:

$$\sum_{l \in FTG_j} (\eta_l B_l)^\beta = \sum_{l \in FTG_j} (A_l)^\alpha$$

(32)

Therefore from (31) and (32) we can say that:

$$\sum_{l \in FTG_j} (B_l)^\beta > \sum_{l \in FTG_j} (\eta_l B_l)^\beta$$

(33)

or,

$$\sum_{l \in FTG_j} (B_l)^\beta > \sum_{l \in FTG_j} (C_l)^\beta$$

(34)

In this paper $\beta$ has been taken as the priority variable of the cost functions, which leads the network to the uniform energy dissipation criterion. Here $\beta$ directly emphasizes the priority of ‘remaining energy of the next node’ variable in the cost functions. $\beta$ used as the exponent of ‘remaining energy of the next node’ variable instead of multiplier and with its value greater than the value of $\alpha$, it can be said that the conservation of uniform energy dissipation criterion will be greater. Since in case of Type III cost function the priority variables are used as multiplier while in case of Type IV cost function the priority variables are used as exponent, we can say that uniform energy dissipation will get higher priority in case of Type IV function with respect to Type III function. For that reason, we can say that the standard deviation of $\{B_1, B_2, B_3, \ldots, B_l\}$ will be higher than the standard deviation of $\{C_1, C_2, C_3, \ldots, C_l\}$. In both cases, one of the entities of any set is zero and in case of $\{C_1, C_2, C_3, \ldots, C_l\}$ the standard deviation is low; therefore, we can say:

$$\sum_{l \in FTG_j} (B_l)^\beta > \sum_{l \in FTG_j} (C_l)^\beta$$

So, we can say:

$$\sum_{l \in FTG_j} (e^x_{rem})^\beta_{III} > \sum_{l \in FTG_j} (e^x_{rem})^\beta_{IV}$$

With this order of complexity and the comparative study of different types of cost functions, we can summarize our conclusion in the Table II.

### VI. Tradeoff Between Total Energy Dissipation and Uniform Energy Dissipation by Using Dynamic Priority Variable

In case of cost calculation, if the value of $\alpha$ is greater than the value of $\beta$ that means we are giving a higher priority to total energy saving. As a result, the time of the last node death will increase. On the contrary if the value of $\beta$ is greater than the value of $\alpha$ then though uniform energy dissipation criteria will be satisfied; total energy saving will be low. This implies that time of last node death of the network will decrease. Our intension is to design a cost function for a network which can extract the maximum lifetime out of that network in terms of first node death and last node death in the network.

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<tr>
<td>Maintaining uniform energy dissipation criterion</td>
<td>Best</td>
<td>Not Applicable</td>
<td>Worst</td>
<td>Medium</td>
</tr>
<tr>
<td>Computational Complexity</td>
<td>Best</td>
<td>Not Applicable</td>
<td>Best</td>
<td>Medium</td>
</tr>
<tr>
<td>Computational ambiguity</td>
<td>Not Applicable</td>
<td>Not Applicable</td>
<td>Ambiguous when equation (29) is true</td>
<td>Ambiguous when equation (30) is true</td>
</tr>
</tbody>
</table>
A. Advantages of varying priority variable with respect to current node energy dissipation

Initially when every node has maximum energy, cost functions will give priority to total energy saving constraints, which will prolong the last node death. But as nodes begin losing energy, priorities also change. The decrease in total energy of a node also decreases the priority of the parameter $e_j$, but increases the relative priority of $e^{\rho}_{rm}$. Thus, the value of either $\alpha$ or $\beta$ may be varied, while keeping the value of the other, constant. The computational cost of Type III cost function is lowest among the four types of cost functions. However, in case of uniform energy dissipation criteria, Type III function performs the worst. If the value of priority variable in case of Type III cost function can be varied, then the life time of WSN with respect to first node death as well as last node death will increase. We can take the value of $\alpha$ as:

$$\alpha = m \left( \frac{e^j_{rem}}{e_{max}} \right)$$  \hspace{1cm} (36)

Here, we take the value of $\beta$ as constant:

$$\beta = m/2$$  \hspace{1cm} (37)

From (36) it can be said that the maximum value of $\alpha$ is $m$ and the value of $\alpha$ is varied as:

$$0 \leq \alpha \leq m$$  \hspace{1cm} (38)

Initially the value of $\alpha / \beta$ is equal to 2. Gradually the value of $\alpha$ will be decreased with respect to $\beta$ and at some point, the ratio $\alpha / \beta$ would be equal to one. Thereafter, the value of $\alpha$ will be decreased further and the ratio will be less than one and finally the value will be zero. When the remaining energy of node $j$ is less, then from (27) it can be said that the value of $\alpha$ will also become very low or very near to zero.

Then from (27) we can ignore the factor $\frac{\alpha}{\beta}$ and $\sum(kd^n_{jl})$ with respect to $\sum_{l \in FTG_j}(e^l_{rem})$. The expression for the remaining energy of node $x$ in case of Type III function is:

$$e^x_{rem} \leq \beta \left( \sum_{l \in FTG_j}(e^l_{rem}) \right)$$  \hspace{1cm} (39)

If the sensor nodes are uniformly distributed over the network, then the value of $e^x_{rem}$ becomes zero, that is, it can be assumed that the remaining energy of the current node also becomes low. Therefore, the relative decrease in value of $\alpha$ with respect to $\beta$ leads to increase in the priority of uniform energy dissipation criterion. As value of $e^x_{rem}$ approaches zero we can say the value of $\sum_{l \in FTG_j}(e^l_{rem})$ also becomes low. Since the value of $\sum_{l \in FTG_j}(e^l_{rem})$ becomes low, then it can be said that the value of $e^x_{rem}$ also becomes low. Therefore, in case of modified Type III cost function, both total energy saving and uniform energy dissipation get high priority without violating each other.

VII. COMPARISON OF PRIORITIZATION VARIABLES ($\alpha$ AND $\beta$) FOR GETTING OPTIMUM ENERGY DISSIPATION

From the above discussion, it is apparent that the relative difference between $\alpha$ and $\beta$ will decrease with decreasing remaining energy of current node. Therefore, the initial value of $\alpha$ will be greater than the value of $\beta$.

$$\beta = m \rho \left( 1 - \frac{e_{rem}^j}{e_{max}} \right)$$  \hspace{1cm} (40)

If we take the value of $\beta$ as per equation (40) and if we keep the value of $\alpha$ constant, then the ratio $\alpha / \beta$ will never be zero. Therefore, from (26) we can say that the Type III cost function will never follow the absolute uniform energy dissipation criterion. When the value of $e^x_{rem}$ is zero then from (26) we can say $\sum_{l \in FTG_j}(e^l_{rem}) = \frac{\alpha}{\beta} \sum_{l \in FTG_j}(kd^n_{jl})$. Therefore, it can be said that when the value of $e^x_{rem}$ is zero then $\sum_{l \in FTG_j}(e^l_{rem}) \neq 0$. From previous discussion, it is clear that if we take $\beta$ as the variable and $\alpha$ as constant then uniform energy dissipation criteria are violated a little bit. On the contrary it can be said that if we take $\alpha$ to be variable and $\beta$ to be constant then uniform energy dissipation criteria is maintained.

VIII. RESULTS

Experiments have been carried out to provide evidences in support of the proposed mathematical model for different types of cost functions. Here the energy dissipation model described in [19][20] is used. As per,

$$E_{TX} (m,d) = m^*E + m^*\eta^*d^2 \ , \ E_{RX} (m) = m^*E \ .$$

Where

$$\begin{align*}
E_{TX} & = \text{Energy consumed for transmission of message}, \\
E_{RX} & = \text{Energy consumed for receiving message}, \\
d & = \text{Euclidian distance between the transmitting and receiving nodes}, \\
\eta & = \text{Permittivity of free space}, \\
m & = \text{Number of bits per packet of a message}, \\
E & = \text{The amount of energy required to receive one bit of message}.
\end{align*}$$

We consider $E = 50nJ \ , \ E_{tx} = 50nJ / \text{bit}$. If we consider the transmission range of sensor node to be in the range 150-300 meter [21], then we can say: $E \ll \eta^*d^2$. According to the
value of $E$, $n$ and $\eta$ we can ignore the factor $m \cdot E$ in case of
sending a message. For the purpose of simulation, we assume
the following network parameters as listed in Table III.

<table>
<thead>
<tr>
<th>Name of Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of packets per message</td>
<td>8</td>
</tr>
<tr>
<td>Number of bits per packet</td>
<td>200</td>
</tr>
<tr>
<td>Maximum energy per node</td>
<td>500000000 nJ</td>
</tr>
<tr>
<td>Area of network</td>
<td>1.5x1.5 Sq Km</td>
</tr>
<tr>
<td>Total number of nodes in the network</td>
<td>361</td>
</tr>
<tr>
<td>Transmission range</td>
<td>120 meters</td>
</tr>
<tr>
<td>Permittivity constant of medium ($\eta$)</td>
<td>100 nJ / bit / m$^2$</td>
</tr>
</tbody>
</table>

Simulation results are presented in Fig. 2 to Fig. 13. We have
simulated different types of transmission cost functions (Type I-Type IV) on for the AUEDDD algorithm. Lifetime is
considered to be the total number of messages transmitted in
the network till the first node death. Fig. 2 shows the changes
in life time by adopting different types of transmission cost
functions for selecting next node in case of single sink network. Also, Fig. 3 shows the changes in life time by
adopting different types of cost functions for selecting next
node in case of multiple sink network. In case of both types of
network arrangement, with increase in the value of $\beta$ the life
time of network will increase using the Type I cost function.
In case of Type II cost function there is no effect in changing
the value of $\beta$. As per equation (12) in case of Type II cost
function the probability function for selecting the next node is
independent of priority variables ($\alpha$ and $\beta$). Therefore, Type
II cost function will not show any variation with respect to
different values of priority variables. Whereas life time will
decrease in both single and multiple sink networks in case of
Type III cost function. But in case of Type IV cost function,
the network life time increases with the increasing value of $\beta$ in case of single sink network but the scenario is just
opposite in case of multiple sink network.

![Fig. 2. Changes in life time by adopting different types of path cost equation in case of single sink network](image)

![Fig. 3. Changes in life time by adopting different types of cost function in case of multiple sink network](image)

The changes in total remaining energy at the end of network
life time for different values of $\beta$ by adopting different types
of path cost equation in case of single sink network is shown
in Fig. 4. Fig. 5 shows the changes in total remaining energy
in case of multiple sink network. If we apply Type III cost
function (equation (9)) then with increase in the value of $\beta$
the priority of uniform energy dissipation will decrease and for
that reason total remaining energy increases in case of both
network scenarios. For Type II cost function the total
remaining energy remains same for different value of $\beta$.
Whereas for the Type I cost function, total remaining energy
at the point of first node death increases by increasing the
value of $\beta$ in both types of network set up (Single and
Multiple Sink). But in case of Type IV cost function the
scenario is different. In case of Type IV cost function total
remaining energy decreases with increase in the value of $\beta$.
However, total remaining energy increases with increase in the
value of $\beta$ in case of Type IV cost function.

![Fig. 4. Changes in total remaining energy by considering different types of transmission cost function in single sink network](image)

![Fig. 5. Changes in total remaining energy by considering different types of transmission cost function in multiple sink network](image)

The change in average energy consumed by a node at the time
of first node death for different values of $\beta$ by adopting
different types of equation in case of single sink network and
multiple sink network, is shown in Fig. 6 and 7, respectively.
Fig. 6 says average of per node energy consumption will be
higher for higher value of $\beta$ by applying Type I and Type IV
cost function in case of single sink network. For Type II cost
function average per node energy consumption will remain
same for single sink network. In case of Type III cost function,
average per node energy will decrease with the increase in
value of $\beta$. Fig. 7 also shows that Type II cost function will
not show any changes with changing value of $\beta$. But in Fig.
10 average per node energy consumption shows very little
change and hence the graph looks like a linear graph. The
change in average per node energy dissipation monotonically
increases with the increase in the value of $\beta$ in Type I cost
function. While Type IV cost function is adopted for data
transmission, average life time will decrease with the increase
in the value of $\beta$. 

![Fig. 6. Average per node energy consumption in single sink network for different types of transmission cost function](image)

![Fig. 7. Average per node energy consumption in multiple sink network for different types of transmission cost function](image)
We have simulated the type I path cost function by considering variable beta ($\beta$) in case of multiple sink and single sink network. The value of alpha ($\alpha$) is constant (10) in these simulation experiments, which can as well be varied. In the simulation experiments, $\beta$ is considered to be (i) having a static (constant) value and (ii) varying as well. Fig. 8 – Fig. 11 show the scenarios. Fig. 8 and Fig. 9 show that by varying beta ($\beta$) accordingly, life time decreases little bit whereas Fig. 10 – Fig. 11 show that average energy consumption will also decrease at the time of first node death in the network. Therefore, if we vary beta, then life time will decrease and average energy consumption will also decrease, meaning total remaining energy will be more in case of variable beta which will prolong the network life time with respect to average node death in the network.

Table IV and Table V show the performance of different cost functions with respect to different parameters for maximizing life time of the network. Table IV presents the performance of different cost functions in case of single sink network, whereas Table V presents the performance of different cost functions in case of multi sink network. In Table IV and Table V, performance is represented by $P_i$ where $i \in \{1, 2, 3, 4\}$.

With respect to any parameter having a pair of value $P_i$ and variable beta

Both the Figures 12 and 13 show that if beta is varied, then percentage of average energy consumption will get reduced more than percentage of the changes (reduction) in life time of the network. Therefore, we can say if we adopt variable beta concept, then we can save significant amount of energy.

**Fig. 12 and Fig. 13 show comparison of the percentage of changes between life time and average energy consumption at the time of first node death by changing the value of beta for single sink and multi sink simultaneously.**

The equation we have followed to find out the changes is:

$$\% \text{ of change} = \frac{(\text{value of parameter for static beta} - \text{value of parameters for dynamic beta}) \times 100}{\text{value of the parameter for static beta}}.$$
$P_j$ for any two cost functions where $i$ is greater than $j$ the cost functions which has the value $P_j$ shows better performance than other. It can be concluded that Type I cost function gives best result and Type III cost function performs worst in most of the cases. Only in case of Single Sink network, Type III cost function performs better than Type IV. In most of the cases the order of performance with respect to all the parameters in an ascending order is as follows: TypeIII < TypeIV < TypeII < TypeI.

Table IV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type I Cost Function</th>
<th>Type II Cost Function</th>
<th>Type III Cost Function</th>
<th>Type IV Cost Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life time of WSN</td>
<td>P1</td>
<td>P2</td>
<td>P3</td>
<td>P4</td>
</tr>
<tr>
<td>Total remaining energy</td>
<td>P1</td>
<td>P2</td>
<td>P4</td>
<td>P3</td>
</tr>
<tr>
<td>Average per node consumed energy</td>
<td>P1</td>
<td>P2</td>
<td>P4</td>
<td>P3</td>
</tr>
</tbody>
</table>

Table V

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type I Cost Function</th>
<th>Type II Cost Function</th>
<th>Type III Cost Function</th>
<th>Type IV Cost Function</th>
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<tbody>
<tr>
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<td>P4</td>
<td>P3</td>
</tr>
<tr>
<td>Average per node consumed energy</td>
<td>P1</td>
<td>P2</td>
<td>P4</td>
<td>P3</td>
</tr>
</tbody>
</table>

The simulation result shows that Type IV cost function gives worst performance while the theoretical analysis shows that Type IV gives performance equivalent to Type I cost function where Type I cost function is the best among four cost functions. Intuitively we can say that if we multiply Type IV cost function by minus one (-1) then its performance will be equivalent to Type I cost function. Therefore we may consider Type III cost function to be worst performing cost function.

From expression (24) we can see the term $\left( e_{avg}^\beta - (kd_{avg}^n)^{\frac{1}{\alpha}} \right)^{\frac{1}{\beta}}$ at the right-hand side. The term $\left( e_{avg}^\beta - (kd_{avg}^n)^{\frac{1}{\alpha}} \right)$ will be a fractional value. If we increase the value of beta then the value of $\left( e_{avg}^\beta - (kd_{avg}^n)^{\frac{1}{\alpha}} \right)^{\frac{1}{\beta}}$ will increase while the value will decrease with decrease in the value of beta. Therefore, while the value of beta is lower, then from the above analysis and expression (24) we can say the value of $e_{rem}^\beta$ will be lower and as a result, the nodes of the network will dissipate energy more uniformly. Therefore, while the value of beta is lower, then lifetime will be higher in case of Type IV cost function.

X. CONCLUSION

In WSNs, the life time of the node which dies first is considered as the life time of network. Therefore, the motto is to increase the life time of the node which will die first. This paper studied different types of cost functions on top of AUEDDD algorithm. Initially, we have analyzed different cost functions theoretically. We considered 4 possible types of cost functions for the analysis. Theoretical analysis shows Type I cost function performs best though the computational complexity is highest for the function. Here, Type II cost function is basically a special type of Type I cost function where the value of all priority variables is 1. Type III cost function performs opposite with respect to other cost functions. By increasing the value of $\beta$ the message transfer cost will decrease in case of Type III cost function whereas for the other types of cost functions, the cost will increase. The AUEDDD algorithm[3] has been simulated using similar simulation environment (Matlab tool) and similar simulation parameters. Simulation results also show similarity with theoretical analysis and Type I cost function performs best amongst other types of cost functions. This paper also proposes dynamic priority variables for getting better result. By adopting these, a network can take care of first node death and the total energy consumption at the time of the first node death. Although the time of the first node death is considered to be the life time of any WSN, yet after the death of the first node, the WSN remains alive with the help of rest of the nodes. Until the first node death, the WSN will work flawlessly. The theoretical analysis and simulation result says that if we adopt dynamic cost functions then we can do better tradeoff between uniform energy dissipation and total energy saving of the network, which leads to tradeoff between life time of the node which will die first and the average life time of all nodes. The nodes which reside nearer to the sink will dissipate more energy than the other nodes which are away from the sink node. In future, our work will concentrate on efficient node deployment policies and analysis of different cost functions. Although Type I cost function is claimed to be the optimum function, other functions may work better than Type I cost function where energy dissipation is not a constraint. For example, Type III cost function will work better than Type I cost function where battery life time is not an issue (Health Care Monitoring) because the time complexity of Type III cost function is less than Type I cost function.

REFERENCES


