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Route Scheduling for HSSP using Adaptive Genetic Algorithm with Constructive Scheduling Technique

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Abstract—Shortest path finding has been a challenging task in most of the complex multipath scenarios. The complexity rises with the introduction of constraints to the scenarios. Healthcare service to the patient is one of the real world problems where travelling path has significant impact on the service time. The purpose of this research is to develop new approach to solve multiple travelling salesman problem (MTSP) for healthcare staff members offering healthcare services at patients homes travelling in different routes with the minimum total cost. The proposed approach uses Genetic Algorithm (GA) combined with Constructive Scheduling, Local Search, and Adaptive Technique to increase the efficiency. A case study with 45 patient task locations is generated according to referenced work. The result shows that the combined algorithms explore improved solution than that of the traditional GA. Constructive Scheduling using K-mean algorithm is applied to generate initial chromosome which provides improved results with acceptable computational time. Also, Adaptive GA shows a few different solutions to the traditional GA. All these approaches are beneficial to the traditional method in shortest path finding problems.

I. INTRODUCTION

Nowadays, there is a trend to allow the older people to stay at home as long as possible rather than having to shift into the retirement home. This trend is growing in many countries because of the issues like rise in life expectancy. By 2035, the number of people aged over 65 in British families will reach at approximately 4 million[1] and the number of British families with some members over 50 has increased by over one million since 2008 [2]. Because of such issue, there are many private and public organizations that are providing care delivery for elders. An effective scheduling of the healthcare in such situation is important. This problem is called Healthcare Staff Scheduling Problem (HSSP) which provides several kinds of healthcare services such as nursing, physical therapy, speech therapy, medical social services, medical visits, house cleaning, home life aides, and old people assistance [3]. In the UK, when senior citizens need health care services, it is the duty of local councils to provide qualified staff to service them at home. To solve health care problem for elders, many researchers have proposed different approaches. Most of them mainly focus on as optimizing the shortest path with only a single requirement task. However, those approaches can be improved. Therefore, we propose an improved scheduling algorithm for solving HSSP using combined approach based

on Hybrid Heuristic, Evolutionary Algorithms, Local Search Techniques, Constructive Scheduling, and simulation technique. The major objective is to design an efficient schedule under specified time and capacity constraints.

The purpose of this research is to solve HSSP using three-step scheduling technique by dividing the problem into different sub-problems and then find solutions with these steps for route scheduling, resource selection, and local improvement: Routes scheduling focuses on how to arrange effective routes for staff with minimum distance travel time and travel cost, Resource selection points to match qualified staff to each route with the minimum cost and the preferences by the scheduler and also a customer representative under feasible time constraint. The Local Improvement enhances output solution generated by the resource selection using swapping task based on the cost function.

In this paper, we focus on Routes scheduling which has two objectives. The first objective is to minimize the shortest path for many staff offering healthcare services at patient homes in different routes. Constructive Scheduling using K-mean algorithm to generate initial chromosome instead of random generation technique is applied to explore improved solution. The second objective is to minimize computational time of execution. The paper is organized with following sections. A literature review is written in Section 2. Descriptions of HSSP including problem details, Constructive Scheduling and Adaptive GA are explained in Sections 3. Section 4 provides the result of all scenarios, followed by the computational time of experiments in Section 4. Finally, the conclusion and the direction for future works are presented in Section 5.

II. LITERATURE REVIEW

Several methods for nurse scheduling problems have been proposed. Felici and Gentile [4] established mathematical model in forms of an integer programming model that maximizes the total satisfaction of the nursing staff. Bard and Purnomo[5] proposed a mathematical programming model for minimizing the penalty of nursing staff members violating preferences. They adopted the column generation scheme to solve the problem. Apart from pure mathematical model, computational concept in term of fuzzy was applied to multi-objective integer programming model in a view of changeable

factors that affect nursing timetable and preferences [6]. In a health-care unit and adopted case study, a mixed-integer programming model for the nurse scheduling problem was implemented. The result showed an improved schedule than the nurse schedules planned manually [7]. Smet et al.[8] solved the nurse scheduling problem using a generic mathematical model, which was related to the common elements used in antecedent works and some limitations that were usually ignored in the medical center. Maenhout and Vanhoucke[9] created a linear integer programming model to support planning for nurses allocated to different departments in a hospital under conditions of staff appointment, nursing shift in particular ward policies and personality traits. Fan et al., [10] used a binary integer programming model to plan a practical solution of management in nursing timetable. The purposes were to maximize all nurses satisfaction after considering seven shifts of both 8.5 and 12.5 hours a day and some hard and soft restrictions. Evolutionary algorithm population-based learning has become more popular in nurse schedule management. An electromagnetic algorithm combined three local search methods for variable neighborhood was used to cope with the nurse scheduling problem by Maenhout and Vanhoucke [9]. Bai et al.[11] proposed a hybrid evolutionary algorithm with the local search associated with simulated annealing hyper-heuristic with the better result. Hadwan et al. [12] proposed a harmony search algorithm for the nurse scheduling problem for a hospital in Malaysia. Todorovic and Petrovic [13] dealt with the difficulty of nurse scheduling using a bee colony optimization which is able to eliminate some ineffective plan from the neighborhood solution. Constantino et al. [14] proposed a new deterministic heuristic algorithm to solve a nurse scheduling problem consisting of two phases; Constructive scheduling phase and improving phase. Gao and Lin [15] developed a mathematical model to overcome the nurse scheduling problem. The goal of research was to find the maximum happiness level while nurses were working considering hospital regulations and then they used classical PSO to solve the problem to decrease the time-consuming manual scheduling. In the real world problem, an algorithm based on the vertebrate immune system compared to Multi-objective GA was proposed to handle with violated restrictions by Maenhout and Vanhoucke[9]. The reproduction of experimental showed promising results for coping with the nurse rescheduling problem. Moreover, an evolutionary algorithm and a branch-and-price approach were combined to reschedule planning to solve violated constraints for nurses from external department by Wright and Vanhoucke [16]. To solve larger HSSP, some researchers proposed a hierarchy problem solving procedure to divide the problem into smaller sub-problems in diverse domains in order to extend the search space and find the global optimal solution[17]. In addition, Immigrant Scheme, a local improvement, was developed to achieve a better result in comparison with the traditional genetic algorithm. The immigrant scheme is designed to recall the best solution or population stored in memory when the new return output is lowered continually[18].

III. PROBLEM DETAILS

In this section, HSSP is explained and also the new modified route scheduling approach using Constructive Scheduling technique is described as follows:

A. Problem description

The problem description given below is derived and adopted from referenced papers [19] for HSSP.

- Staff members (care takers) are assigned jobs according to patient requirement given Table I.
- Each job has the same priority. Patients cannot demand specified service time like e.g. 8.00 a.m. or 1.00 p.m. because it is free service from local council.
- Each care worker starts from Home care office at 8.00 a.m. and each job must be completed within assigned time windows. Also, Operating time has been investigated in the research. Default value is an hour per service. Thus, each route contains approximately 5 tasks each day.
- The location of patient homes is defined by Geolocation (Latitude and Longitude coordination).
- The travel speed depended on modes of transportation for care workers comprising of public transportation with taking train, subway and public buses, and walking.
- We focus on physical therapists only in this research.

B. Proposed techniques: Route scheduling

The goal of route scheduling is to create routes for staff members to service patients at different task locations with the total shortest path based on Genetic Algorithm. This is one of the most popular searching algorithms as it is a suitable global solution technique for complex problem without applying excessive complex mathematical model. GA processes a structure using chromosome representation and randomized operators to evolve solution [20]. Problems are encrypted as chromosome which represents possible solution before using crossover and mutation operator to create new offspring.

In this work, the multiple traveling salesman problems (MTSP) chromosome representation is designed to develop for experiments. It provides multiple routes or salesmen within a single chromosome which complies with HSSP. The number of salesmen is referred to the number of routes collected to one chromosome representation. In Fig. 1 and 2, a chromosome is designed to contain three routes($n\text{-route}=3$) where Route-1 starts from the Health Care Office going to $Task_A$, $Task_B$, $Task_C$, $Task_D$, and then go back to the office at the end. Route-2 begins from the office going to $Task_D$, $Task_K$, $Task_G$, $Task_L$ and going back to the office like the first route. The last one is route-3 which starts at the same Health Care Office and going to $Task_B$, $Task_J$, $Task_F$, $Task_H$, and then going back to the home office.

C. Initial population generation

1) *Random chromosome*: random chromosome is a classic chromosome creation approach which is widely used to generate initial random solution for solving HSSP. This approach

Fig. 1. 12 bits MTSP chromosome representation with n-route =3

Fig. 2. MTSP chromosome representation with n-route =3

guarantees that the chromosome represents a solution within the possible search space.

2) *Constructive Scheduling with K-mean*: According to the paper[21], researchers reported that using Constructive Scheduling at the beginning of experiments which provided the improved solution. Even though such approach shows conspicuous solution, it is difficult to implement in the real circumstance. Thus, manual Constructive scheduling is almost fabulous to utilize in the real situation. In this research, new constructive technique with K-mean clustering algorithm [22] is proposed to partition the locations of patient homes into k sub-groups or k-routes where K is the number of clusters. This clustering technique is based on the centroid values and each centroid tries to combine closer task location in equation (1) and (2).

$$J(C_k) = \sum_{x_i \in C_k} \|x_i - u_k\|^2 \quad (1)$$

$$J(C) = \sum_{k=1}^K \sum_{x_i \in C_k} \|x_i - u_k\|^2 \quad (2)$$

The major steps of K-mean algorithm are as follows:

- 1) Select an initial centroid with K clusters; repeat steps 2 and 3 until cluster membership stabilizes.

Fig. 3. An example of random chromosome conduces to an inferior route

- 2) Generate a new partition by assigning each pattern to its closest cluster centre.
- 3) Compute new centroid of each cluster.

In Figure 4 and 5, Constructive scheduling function is created to store key variables consisting of Input array or random chromosome containing a task sequence and **MaxTaskEachRoute-array**. This array is declared to control the limited number of tasks each route. Next, initial centroids are selected randomly and then these numbers are filled up output array at first index of all three routes (T2 for route1, T1 for route2, and T9 for route3).

At the second step, all centroids are updated respectively by finding the next closest task location to each route. For the first centroid, the nearest task is T12 at (5.0, 7.5) and then new centroid1 is (4.25, 7.0). The second centroid is combined with T6 and then is moved to new position at (7.75, 8.75). The last centroid has T13 to be the nearest task location at (6.0, 1.0) and new position of centroid3 is (6.0, 2.25). The process is repeated until output chromosome is full as shown in Fig. 4.

D. Crossover and mutation operator

- 1) *Partiallymapped crossover (PMX crossover operator)*: the crossover rate P_c is defined as the ratio of the number

Fig. 4. Using K-mean for initial chromosome generation

of genes that will be crossoverd to the population size which ranges from 0% up to 100%. In Fig.5, P_c is set to 5% that is approximate 2 bits to crossover as shown in step I. This operator creates new offspring by choosing a subsequence of a tour from one chromosome and keeping the order and position of as many tasks as possible from another chromosome. A subsequence of a tour is selected by selecting two random cut points. In this example, Bit_{12} and Bit_{13} of each chromosome are selected (step II.) and exchanged values between chromosomes (step III.) before repeating to fill up values in empty bits (T13 and T2 in row1 and T7 and T10 in row2) (step IV.) to generate new chromosomes (step V.).

Fig. 5. 2 bits crossover

Fig. 6. 2 bits mutation

2) **Mutation**: this operator flips or alters one or more bit values randomly in a chromosome. The values of the selected genes are then swapped between tasks. The mutation rate P_m is defined as the ratio of the number of genes that will be mutated to the population size ranging from 0% up to 100%. The mutation operation also assists the exploration for potential solutions which may be overlooked by the crossover operation in Fig. 6.

E. Adaptive GA

There are two essential parameters which are controlled in GA: crossover rate P_c and mutation rate P_m . Technically these parameters are set as constant before running experiment. These numbers affect significantly the quality of reproduction conducting to a variety of computational time. Adaptive parameter control is one of parameter control techniques apart from fixed parameter rate which allow adjusting parameters while running GA using feedbacks from the fitness values [23]. In this research, GA with adaptive crossover rate (P_c) and

Parameter list	Level of parameters					
Crossover rate: P_c	5	10	15	20	25	30
Mutation rate: P_m	5	10	15	20	25	30
Constructive Scheduling:CS	Enable			Disable		
Adaptive GA	Enable			Disable		
Local Search	Enable			Disable		
The number of patient locations	45					
The number of GA population	4					
Maximum iteration	250					
Replication	3					

TABLE I
PARAMETER SETTING

adaptive mutation rate (P_m) is developed based on equation (3) and (4) respectively.

$$P_{m(new)} = \begin{cases} \frac{P_{m(old)}+1}{2} & fitness_{current} > fitness_{old} \\ P_{m(old)} & , otherwise \end{cases} \quad (3)$$

$$P_{c(new)} = \begin{cases} \frac{P_{c(old)}+1}{2} & fitness_{current} > fitness_{old} \\ P_{c(old)} & , otherwise \end{cases} \quad (4)$$

According to equation (3) and (4), P_c and P_m have been decreased when running iterations grow up. Consequently, the new solution of each generation has been changed slowly to reach the optimal solution.

F. Experimental design

There are nine parameters, given in Table 1, are used in all experiments with Constructive adaptive GA with Local search. First five of parameters P_c , P_m , CS, AdaptiveGA, and LocalSearch are adjusted to create 288 possible scenarios ($6P_c * 6P_m * 2CS * 2AdaptiveGA * 2LocalSearch$). The remaining parameters (N-location, N-population, Maximum iteration, and Replication) are fixed. The total number of runs is 864 (288 scenarios*3 replications). The maximum number of iterations was set to 250 for each run. Results of all scenarios are calculated to find average fitness value and average computational time.

G. Dataset

Dataset includes six fields consisting of task number, x-coordination, y-coordination, operating time, staff requirement and type of locations (patient location and home office location). Euclidean distance is implemented according to the paper [4] that used two dimensional symmetric problems.

IV. RESULTS

The proposed algorithms have been implemented in R programming language on Intel i5 system with 8 GB RAM, running at 3.3 GHz under Microsoft Windows 7 64-Bit. All of scenarios are generated from P_c and P_m from 5% to 30%. Simultaneously, Adaptive GA, Constructive Scheduling and Local Search functions are utilized to create the scenarios. PMX crossover technique has been used for 8 major sections as follows:

- 1) Adaptive GA, Constructive Scheduling and Local Search are disabled from scenario 1 to 36

- 2) GA with Local Search from scenario 37 to 72
- 3) Adaptive GA from scenario 74 to 108
- 4) Adaptive GA with Local Search from scenario 109 to 144
- 5) GA with Constructive Scheduling from scenario 145 to 180
- 6) Adaptive GA with Local Search from scenario 181 to 217
- 7) Adaptive GA with Constructive Scheduling Local Search from scenario 218 to 252
- 8) All algorithms are enabled from scenario 253 to 288

Experimental results are shown in terms of fitness value in Fig.7 in terms of computational time in Fig.8 and computational time of 281 scenarios in Fig 9. The first objective function of this experiment is to minimize the total distance of all routes. The constraints are used in a way that the staff must visit each task location and each route contains five locations. The total distance will increase when task locations are assigned into each route. Fig.7 provides result of fitness value in terms of average total distance containing all routes for care staff member offering healthcare service during three runs on each of scenario.

In Fig.7, the line graph shows the average total distance of 288 possible scenarios used to run the experiment. To begin with a first point (all algorithms are disabled), the total distance begins at the high level at 206.64 units and then such distance significantly goes down steadily to 182.23 units at scenario 73 while P_c and P_m have been decreased. From scenarios 74, Adaptive GA and Local Search are utilized. The total distance continues to fall steadily before dropping sharply at scenario 145 with 143.71 due to the fact that Constructive Scheduling begins to be used while Adaptive GA and Local Search are inactivated. After that, the total distance is fluctuated gradually from scenario 146 to 288 while P_c and P_m have been gently adjusted. The lowest total distance is at scenario 285 of 134.74 units which all algorithms including Constructive Scheduling, Adaptive GA and Local Search are used again with P_c and P_m are set at 5% and 20% respectively.

Fig. 8 illustrates the fitness values of scenario 285 which is the best solution of all runs. At the beginning, the total distance value begins at around 250 distance units before it decreases remarkably until iteration 111. Between iteration 112 to 173, there are some variations but that has a decreasing trend before it decrease slowly from iteration at 166 to the end while the lowest fitness point is at iteration 223 with approximately 134.74 distance units. The scheduled routes are shown in Fig.9. We can see that task locations of each route are in the same area.

The second goal included in this research is to minimize computational time. Even though the fitness value is decreased when we use complex algorithm, it is not appropriate to use in the real circumstance due to the fact that it consumes higher execution time. Thus, trade-off analysis for both objectives is necessary inevitably. Suggested algorithm has fast speed for solving and generates improved solution at the same time. It can be seen from the data that the average computational

Fig. 7. Total distance of 288 scenarios

Fig. 8. The fitness values of scenario 285

time increases remarkably at approximately 3.77 seconds when Local Search is enabled between scenario 73 to 144. From scenario 145 to 180, Constructive Scheduling is used only. It decreases to around 2.79 seconds before reaching a peak at around 4.21 second in scenario 181. It drops slightly as P_c and P_m are adjusted slowly from 30% to 5%. At scenario 217, the computational time is in low level due to the fact that Local Search is disabled while Adaptive GA and Constructive Scheduling are used. From 253 to 258, it goes up again between 3.87 and 4.03 seconds. To conclude, the computational time increases when any scenarios use Local Search and it drops gently when P_c and P_m have been changed from high to low level.

V. CONCLUSION

In this paper, new Constructive Scheduling Technique using K-mean algorithm is proposed for solving HSSP in term of MTSP. A new approach shows outstanding improved solution compared with using other algorithms. In addition, GA integrated with other algorithms (CS, Adaptive GA and Local Search) provides the best output solution though the computational time is on higher side due to the fact that Local

Search consumes higher time. Moreover, P_c and P_m barely affect the mode. When P_c and P_m are set in low level, they can explore a few better solutions with less computational time. Considering the objectives, the proposed approach has acceptable computational time with the improved total travel distance minimization.

VI. FUTURE WORK

In this paper, we realized that Local search or 2-OPT consumes a large amount of computing time as the replaceable rate of its algorithm is fixed all the time during running algorithm. To handle with the problem, we will propose Adaptive Local Search which the replaceable rate can be adjusted automatically depending on outputs of GA.

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Fig. 9. Computational time of 281 scenarios

Fig. 10. The best solution at scenario 285 for 45 task locations with 8 routes.

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