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Effect of channel geometry on the performance of the Dethridge water wheel

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Abstract

Dethridge water wheel is a simple hydraulic machine originally invented for measuring volume of flow supplied to the farms. The wheel has been in widespread use for more than a century for the application of water charges in irrigated farmlands. The Dethridge water wheel resembles distinct characteristics making it a suitable device for utilising very low-head sites within irrigation canals, small streams and at the outlets of the waste water treatment plants for pico-hydropower generation. In this paper, performance characteristics of the Dethridge water wheel model is studied in different channel geometry settings. Different wheel to channel width ratios and gradual transition shapes were tested. The wheel performance improves in the channel width that is two to three times greater than the wheel width. The gradual transition shape has however insignificant impact on the performance of the wheel.

Keywords: Pico-hydropower, Dethridge wheel, Channel geometry, Very low-head, Rural electrification

1. Introduction

Most of the very low-head sites for pico-hydropower are within redundant mill sites, weirs, irrigation networks and waste water networks. These sites with existing civil infrastructure, predictable flow rate and useful head represent significant future development potential for low-head hydropower development. Abandoned old mill sites have existing diversion structure and suitable head for very low-head hydropower. Flow in irrigation canals is usually diverted through a diversion canal, have a wide distributary network and are also often equipped with small drops to reduce bed erosion. With predictable and almost constant flow rate, waste water networks are also highly potential for hydropower [1,2]. The outflow discharged into the river at the outlet of waste water treatment plants generally have very low-head difference and almost constant flow rate. These resources offer a considerable scope to harness small scale hydropower [3,4,5,6,7]. However, technology suitable for employing these very low-head resources is still economically challenging.

Recent studies have shown that conventional technologies such as water wheels are suitable devices for very low-head sites [8,9,10,11,12]. Special interests would be for decentralised rural areas where water wheels would constitute economically and ecologically viable source of power generation [13]. The simple design, easy maintenance and ability to easily handle foreign objects make the water wheels an attractive source of power for decentralised rural applications.

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One of the suitable water wheels for very low-head application is the Dethridge water wheel. It was originally invented for measuring volume of flow delivered from the outlets of the irrigation canal to the farm for application of water charges. The wheel has been in widespread use for more than a century in the irrigated farmlands of Australia, to some extent in the USA and in Asian countries [14]. Unlike conventional water wheels, the big hub of the Dethridge wheel acts like a dam and creates a head drop by itself. This distinct characteristic of the wheel could be utilised for hydropower from sites with very low-head differences. The simple and robust design of the wheel makes it even more suitable for its application in rural areas of the developing countries. Physical model tests of Dethridge water wheel have shown an efficiency of around 60% and ample amount of power output that could be utilised for simple applications like lighting, listen to the radio broadcasts, and battery charging facilities [15].

Channel geometry is an important performance and economical criteria to be considered for the implementation of the Dethridge water wheel in practice. The objective of this paper is to study the effect of channel geometry on the performance of the Dethridge water wheel. Tests on the Dethridge water wheel physical model were carried out to identify the optimal wheel to channel width ratio and channel transition shapes. Wheel width ($b$) to channel width ($B$) ratios of 1:1, 1:2, 1:3 and 1:4 were investigated. Furthermore, gradual contraction and expansion profiles on the upstream and downstream of the channel were tested. The physical model is described, the method of measurement, uncertainty calculation and data analysis are presented, and the performance characteristics of the wheel at different settings are compared and discussed.

2. Methodology

2.1. Test rig

The physical model of the Dethridge water wheel is tested in a rectangular flume. The flume is 20 m long, 1 m wide and 1.5 m deep and houses a model of the Dethridge wheel with shroud, shaft, torque transducer, speed sensor, stilling tubes, an inlet tank at the upstream and a control weir at the downstream. Side walls of the flume are made up of a glass and the bottom is a smooth concrete floor. Water was supplied through a pump of 501/s maximum capacity. The general layout of the test rig is shown in Fig. 1.

![Figure 1: Schematic sketch of the test facility](image)

The Dethridge wheel model has a radius ($R$) of 30 cm (Fig. 2). The hub of the wheel is made up of PVC piping, which has a radius $r = 20$ cm and width $b = 25$ cm. The wheel is covered with 20 mm thick PVC
side covers on both sides to ensure the stability of the wheel and to avoid the accumulation of water inside
the hub which would otherwise retard the wheel motion. The gap between the sides of the housing and the
wheel, and the bottom gap are 1 mm. The bottom curved shroud profile makes an angle of $\beta = 70^\circ$ to the
center of the wheel. Six steel blades of 2 mm thickness are mounted along the circumference of the hub of
the wheel. The blades are 10 cm long ($l$) and are bent in V-shape to acquire an angle of $\alpha = 127^\circ$. At the
apex of each blade, an air vent is located to facilitate the filling and emptying of adjacent compartments as
they enter and exit the water surface. The apex of the V is leading in the direction of rotation. Both sides
of the blades are chamfered to match the fillets at the junction of the side walls and the floor. The blades
are painted to reduce the surface roughness and to prevent corrosion. A stainless steel shaft of diameter
20 mm and 45 cm long was used.

![Dethridge water wheel model](image)

Figure 2: Dethridge water wheel model

2.2. Measured variables and methods

Flow rate ($Q$) delivered to the test flume is measured using a magnetic flow meter from Krohne Aquaflux F
with IFC 110 F signal converter. This is a measure of volumetric mass flow rate through the wheel control
volume including the amount of leakage flow through the side and bottom clearance gaps. The total head
($H$) acting on the wheel is the difference in the total heads between the upstream and downstream of the
wheel. The elevation head ($z_1 - z_2$) is 0. Water levels ($h_1$) and ($h_2$) were measured at the immediate
upstream and downstream of the wheel control volume. To measure the water levels, two stilling tubes were
installed and depth gauges were used for the manual reading of the flow depth values. The subscripts 1 and
2 refer to the upstream and the downstream, respectively. The mean velocity of the flow is then calculated
from the known area of the flow ($A_1 = Bh_1; A_2 = Bh_2$) and the flow rate ($v_1 = Q/A_1; v_2 = Q/A_2$), where
$B$ is the channel width. The total head $H$ is therefore given by,

$$H = \left( z_1 + h_1 + \frac{v_1^2}{2g} \right) - \left( z_2 + h_2 + \frac{v_2^2}{2g} \right) = \left( z_1 - z_2 \right) + \frac{h_1 - h_2}{2g} + \frac{v_1^2 - v_2^2}{2g}$$  \hspace{1cm} (1)$$

The shaft torque ($\tau$) is generated as a result of the energy transfer between the fluid and the rotating wheel.
The shaft torque was transmitted to the torque transducer shaft through the Polyurethane synchronous belt
drive. The measured torque therefore included mechanical losses due to the bearings and the belt drive. The
torque on the wheel shaft was measured using the torque transducer from HBM model T22. The rotational
speed ($N$) was measured using a solid shaft pulse encoder of make IFM model RB1015. The speed of the
wheel was varied by applying load on the wheel through a Hysteresis braking system from Magtrol model
HB-140M-2. The brake system operates at a higher speed range so a gearbox from Bretzel GmbH was used
to step up the shaft speed at the brake end. The brake was electrically operated through a power supply

3
of Magtrol make model 5210. Output signals from all the measurement instruments were collected into a junction box and fed to the computer using a LabVIEW based program. For each constellation, data were acquired for approximately one to two minutes and the mean value were taken for analysis. The detailed description of the test rig and measurement system is presented in Paudel [15].

The performance variables were measured for flow rates of 6 to 20 l/s. Beyond this flow rate, the test rig was not capable to accommodate the severe splashing occurred at the upstream by fast revolving wheel. Flow depths \((h_1 = 44.65 \text{ cm})\) and \((h_2 \approx 6.2 \text{ cm})\) were kept constant in order to be able to compare the performance between different settings. The measured data showed that the speed of the wheel and flow rate has a linear relationship given by Eq. 2 at constant flow depths. The constants \(a_1\) (slope) and \(b_1\) (y-intercept) in Eq. 2 are derived from measured results. The flow rates outside of the measured range including the minimum flow rate required to start the wheel into motion and maximum flow rate required for no-load speed are calculated using Eq. 2.

\[
Q = a_1 N + b_1
\]  
(2)

Similarly, the test results also demonstrated that speed and torque have a linear relationship of the form given by Eq. 3 at constant flow depths. Speed-torque line was plotted from the measured data and the constants \(a_2\) and \(b_2\) were determined. Rest of the points on the curve outside the measured range including the stall torque and the no-load speed are calculated using the Eq. 3.

\[
\tau = a_2 N + b_2
\]  
(3)

By applying the Bernoulli’s principle, the work done on the wheel shaft \(W_s\) by the water per unit mass flowing through the wheel is:

\[
W_s = gH - \text{Losses}
\]  
(4)

where, \(H\) is the total head acting on the wheel control volume and is given by Eq. 1. The power delivered to the wheel shaft \(P_{\text{shaft}}\) is the the work done on the wheel shaft times the mass flow rate:

\[
P_{\text{out}} = \rho Q (gH - \text{Losses})
\]  
(5)

Total losses in the wheel control volume comprises of hydraulic losses, leakage losses and mechanical losses (Eq. 6). Hydraulic losses include the fluid friction losses, blade impact losses at the entry, exit losses, as well as complex flow losses within the blade cells such as flow separation, flow circulation and secondary flow. Mechanical losses include losses due to belt drive and the bearings which is constant and assumed to be 5%.

\[
P_{\text{out}} = \rho Q (gH - gH_{\text{hyd. losses}} - gH_{\text{leakage}} - gH_{\text{mech. losses}})
\]  
(6)

Leakage losses are due to the clearance gaps between the wheel and the housing and through the V-shaped air vent. Leakage flow can be quantified from the known volume of the blade cells and the measured speed of the wheel given that the cell compartments are completely filled. Therefore, leakage flow rate \(Q_L\) is given by,

\[
Q_L = Q - V * N/60
\]  
(7)

where, \(V\) is the volume of water occupied in the cells of the wheel which is 38.84 l for this wheel size.
The power output is calculated from the known values of speed of the wheel \((N)\) and the torque \((\tau)\) using Eq. 8. The measured power output \((P_{\text{out}})\) also includes the mechanical losses due to the bearings and the belt drive.

\[
P_{\text{out}} = \omega \tau = \frac{2\pi N\tau}{60} \tag{8}
\]

The shaft power can be measured without information on the losses. The mechanical efficiency of the wheel is therefore the ratio of shaft power output and the hydraulic power input and is given by Eq. 9. This efficiency combines the hydraulic efficiency, volumetric efficiency of the wheel, and the mechanical efficiency of the bearings and belt drive system.

\[
\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\rho Q(gH - \text{Losses})}{\rho gQH} = \frac{2\pi N\tau/60}{\rho gQH} \tag{9}
\]

The analysis of change in performance at different modification stages is done by calculating the relative percentage change in performance variables. Percentage change in the variable of interest is calculated by Eq. 10:

\[
\text{Percentage change} = \frac{\text{Modified case} - \text{Reference case}}{\text{Reference case}} \times 100\% \tag{10}
\]

Uncertainty analysis of the measured data was done using constant odd combination method as described by Moffat [16]. Experimental data have both random (statistical) and bias (systematic or fixed) errors or uncertainty. Systematic errors associated with the measured variables were taken from manufacturers’ specifications. Random error in the measured data was calculated by taking multiple measurements at each constellation. The standard deviation of the mean or the standard error is the measure of random errors in the measurements. The root sum square (RSS) of systematic and random errors gives an uncertainty bound on the measured variables.

The best estimate of the uncertainty in the use of linear regression models for \(N - Q\) and \(N - \tau\) in Eqs. 2 and 3 is calculated by Eq. 11 and 12:

\[
\sigma_Q = \sqrt{\frac{1}{n-2} \sum_{i=1}^{n} (Q - a_1N - b_1)^2} \tag{11}
\]

\[
\sigma_{\tau} = \sqrt{\frac{1}{n-2} \sum_{i=1}^{n} (\tau - a_2N - b_2)^2} \tag{12}
\]

where, \(\sigma_Q\) and \(\sigma_{\tau}\) are the best estimates of uncertainties on the use of Eqs. 2 and 3 respectively, \(n\) is the total number of measurements, \(a_1, b_1\) and \(a_2, b_2\) are the constants for \(N - Q\) and \(N - \tau\) equations (Eqs. 2 and 3) respectively, which were determined from the measured data.

The uncertainties on the main variables were used to calculate the overall uncertainty in derived quantities namely, total head, power output and efficiency given by Eqs. 13, 14, 15 respectively using the method of uncertainty propagation [16]. The dimensionless form of uncertainty on power output and efficiency gives the relative uncertainty bound in power output and efficiency:
\[ \delta H = \pm \left[ (\delta h_1)^2 + (\delta h_2)^2 + \frac{Q^4}{g^2B^4h_1^2} \left( \left( \frac{\delta Q}{Q} \right)^2 + \left( \frac{\delta h_1}{h_1} \right)^2 \right) + \frac{Q^4}{g^2B^4h_2^2} \left( \left( \frac{\delta Q}{Q} \right)^2 + \left( \frac{\delta h_2}{h_2} \right)^2 \right) \right]^{\frac{3}{2}} \]  

\[ \frac{\delta P}{P} = \pm \left[ \left( \frac{\delta N}{N} \right)^2 + \left( \frac{\delta \tau}{\tau} \right)^2 \right]^{\frac{1}{2}} \]  

\[ \frac{\delta \eta}{\eta} = \pm \left[ \left( \frac{\delta Q}{Q} \right)^2 + \left( \frac{\delta H}{H} \right)^2 + \left( \frac{\delta N}{N} \right)^2 + \left( \frac{\delta \tau}{\tau} \right)^2 \right]^{\frac{1}{2}} \]  

where, \( H, \tau, Q, N, P \) and \( \eta \) are the total head, torque, flow rate, rotational speed, shaft power output and efficiency respectively. \( g(= 9.81 \text{ N/m}^2) \) is the acceleration due to the gravity, \( B \) is the channel width, \( h_1, h_2 \) are the upstream and downstream flow depths respectively.

2.3. Channel geometry

The performance of the wheel at different channel widths involved the test with four different wheel width \( (b) \) to channel width \( (B) \) ratios: 1:1, 1:2, 1:3, and 1:4. The model wheel was 25 cm wide and the existing flume was 100 cm wide. Temporary plexiglas walls were installed on the upstream and downstream sides of the wheel to build the desired width ratios.

Transitions on the flow involve significant flow losses due to contraction, expansion and directional change losses. Smooth transition design could minimise significant energy losses at the inlet and outlet of the wheel. Different inlet and outlet forms are tested on the physical model. The contraction transition design on the upstream of the wheel follows a profile recommended by Swamee and Basak [17]. The length of the transition is governed by the side splay of 7:1. Equation 16 is used for the design of the contraction transition.

\[ b = b_0 - (b_0 - b_L) \left[ 1.41 \times \left( \frac{1 - x/L}{x/L} \right)^{1.23} + 1 \right]^{-0.924} \]  

![Figure 3: Channel transition profiles (dimensions are in mm)](image)

(a) Contraction profile  (b) Expansion profile

To design the expansive transition at the outlet of the channel, a profile from Swamee and Basak [18] was taken. Equation 17 gives the optimal profile for the expansive transition. The length of the profile is...
governed by the splay of 7:1. This profile was experimentally tested and found to have the highest efficiency among other tested profiles [19]. The contraction and expansion transition profiles are shown in Figs. 3a and 3b respectively.

\[ b = b_0 - (b_L - b_0) \left[ 2.52 \times \left( \frac{1 - x/L}{x/L} \right)^{1.35} + 1 \right]^{-0.775} \]  

where, \( b_0 = 500 \text{ mm} \) and \( b_L = 1000 \text{ mm} \) are the channel width at \( x = 0 \) and \( x = L \).

3. Results and discussion

3.1. Wheel to channel width ratios

The performance curves at different wheel to channel width ratios (\( b:B \)) is plotted in Fig. 4. Torque, flow rate, total head, power output and efficiency are plotted against the rotational speed of the wheel. Figure 4a shows the torque at different speed of the wheel for all \( b:B \) values. At \( b:B = 1:1 \), the torque shows a significant drop beyond 10 rpm in comparison to the 1:4 width ratio. The no-load speed is reduced from 51 rpm in the case 1:4 to 39 rpm in 1:1 ratio giving a very narrow range of operation. Torque on both wheel to channel width ratio of 1:2 and 1:3 showed improvement over the 1:4 setting with increased no-load speed of 53.4 rpm and 52.8 rpm respectively.

The flow rates for all tested \( b:B \) ratios are shown in Fig. 4b along with the leakage flow rate \( Q_L \) for each case. The leakage flow is calculated using Eq. 7. There is only slight difference in the flow rates between all four cases. At 1:1 ratio, the no-load flow rate reduces to 26.86 l/s from 33.82 l/s in 1:4 width ratio. In 1:3 and 1:2 width ratios the no-load flow rate slightly increased to 33.88 l/s and 34.66 l/s respectively. The leakage flow also shows the similar trend as the flow rates for all four cases.

In Figure 4c, total head for all four \( b:B \) values are shown. In this case, the flow area changes with changing channel width while the flow depth remains the same. The total head in Eq. 1 is therefore a function of the flow rate as well as the channel width. It is clear that the 1:1 width ratio has the highest drop in the head due to the increased velocity head term (negative term in Eq. 1) and vice versa. At higher rotational speeds, i.e. at higher flow rates the difference in total head becomes more obvious.

Power output for all four cases are presented in Fig. 4d along with their uncertainty values. The power output in case of 1:2 and 1:3 both show gain in comparison to the 1:4 width ratio. The 1:1 case shows significant drop beyond 10 rpm. Power output in case of 1:2 and 1:3 width ratios remained same up to \( N = 26 \text{ rpm} \). At higher speed, the 1:3 width shows slight gain. However, uncertainties in power output for both 1:2 and 1:3 case are partly overlapped making it difficult to draw a conclusion.

Similarly, the efficiency of the wheel at different wheel to channel width ratios is shown in Fig. 4e with the uncertainty bound. The efficiency remains highest in case of \( b:B = 1:2 \) above 10 rpm speed. The 1:1 case shows the best efficiency point (BEP) region, the uncertainties are high and partly overlapped. The efficiency of 1:2 and 1:3 width ratios didn’t differ much at higher rotational speeds. The BEP details for all four cases are presented in Table 1.

The effect of wheel to channel width ratio on the wheel performance is studied using the percentage change plots in Fig. 4f. The relative change in performance between 1:2 and the 1:4 width ratios is calculated using Eq. 10. The change in total head is negative and reaches up to \(-11\%\) at highest speed. The change in flow rate is positive for \( N \leq 24 \text{ rpm} \). At higher speed, it becomes negative with maximum change of \(-0.5\%\). The change in power output remains positive through out and shows exponential gain with increasing rotational speed. Despite the decrease in total head, this increase in power output illustrates that the hydraulic losses
are reduced. Change in efficiency is also positive with maximum of up to 5.66% at 38 rpm. This increase in efficiency is attributed to the combined effect of reduced leakage and hydraulic losses.

In Figure 4e, two operating points away from BEP are selected in order to compare the performance of the 1:2 and 1:4 cases. These points correspond to \(N = 8\) rpm and \(N = 30\) rpm respectively. The relative change in head, flow rate, power output and efficiency at these operating points for both cases are summarized in Table 2. At both points, change in power output is positive despite negative change in total head indicating reduced hydraulic losses in case of 1:2 width ratio. Change in efficiency at 8 rpm is positive despite increased leakage losses, i.e. hydraulic losses at this point is dominating. At \(N = 30\) rpm, efficiency shows even higher gain due to the combined effect of reduced leakage and hydraulic losses.

### Table 2: Performance comparison between 1:2 and 1:4 width ratios at different operating points

<table>
<thead>
<tr>
<th>Operating point</th>
<th>(H)</th>
<th>(Q)</th>
<th>(P_{\text{out}})</th>
<th>(\eta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N = 8) rpm</td>
<td>-0.76%</td>
<td>+1.42%+2.91%</td>
<td>+1.25%</td>
<td></td>
</tr>
<tr>
<td>(N = 30) rpm</td>
<td>-4.73%</td>
<td>-0.19%+7.19%</td>
<td>+5.22%</td>
<td></td>
</tr>
</tbody>
</table>

Above results show that the wheel to channel width ratio is an important performance and economical criteria for Dethridge wheel operation. In the 1:1 width ratio, the total head sharply dropped creating a decrease in power output. Moreover, hydraulic losses increased and contributed to the decrease in both power output and efficiency. Flow rates didn’t change much, therefore the leakage losses also remained the same (Eq. 7). The increase in hydraulic losses could be attributed to the high amount of splashing, high amplitude waves on the upstream due to blade and flow interaction; and the amount of air entrainment. At the exit, the water was driven upward in the blade cells. The splashing, size and amount of air bubbles and amount of water driven upward at the exit increased with the increasing speed. The performance of 1:2 and 1:3 width ratios was better in comparison to the 1:4 case. Beyond 12 rpm speed, the 1:2 width ratio showed highest efficiency among others. By having a wheel to channel width ratio of 1:2, i.e. channel width two times greater than the width of the wheel, the efficiency of the wheel can be increased by 2.6% at BEP and at higher speed range to about 5%.

### 3.2. Channel transition shapes

The performance of the wheel with gradual contraction and expansion transition shapes are compared with the performance of the wheel with wheel to channel width ratio, \(b:B = 1:2\), which is the case of sudden contraction from 100 cm channel width to 50 cm on the upstream and sudden expansion on the downstream. In Fig. 5 the torque, flow rate, total head, power output and efficiency against the rotational speed of the wheel are compared for sudden and gradual transition profiles. Overall performance of both settings are discussed using the percentage change in variables between the two cases.

The torque for the gradual and straight transitions are compared in Fig. 5a. There is a very small drop in torque below 20 rpm speed in case of gradual transition. For \(N \geq 36\) rpm, the torque becomes slightly higher than the sudden transition. The no-load speed in gradual transition increases only marginally and reaches 53.45 rpm from 52.8 rpm in case of sudden transition profiles.
The flow rates and the respective leakage flow for both cases are shown in Fig. 5b. Below 24 rpm speed, the flow rate increased in case of gradual transition which means more hydraulic power input is delivered to the wheel for the same rotational speed. The flow rate starts to drop beyond 30 rpm than in sudden transition profile. The flow rate at no-load speed remains almost equal in both cases. The leakage flow decreases with increasing speed and is proportional to the total flow rate through the wheel.

Figure 4: Performance curves at different wheel to channel width ratios
The total head in case of gradual transition profile showed a negligible drop in head due to the increase in flow rate as shown in Fig. 5c. As the flow rate drops beyond 30 rpm, the head starts to increase again since the velocity head is a negative term in Eq. 1. However, the difference remains very small. The flow area at the inlet and exit remains the same for both cases. But due to the change in flow rate, the approach and exit velocity changes creating a difference in the total head.

The power output for both cases are compared in Fig. 5d. The difference in power output between the two settings is negligible. Maximum power output appears at half of the no-load speed in both cases. This insignificant increase in power output at higher speed in case of gradual transition is explained by the increase in head and possible drop in the hydraulic losses. The change in power output lies within the uncertainty bounds in power output.

The efficiency of the wheel for both transition profiles are plotted in Fig. 5e with their corresponding uncertainties. As shown, efficiency in case of gradual transition only starts to increase beyond 22 rpm speed. At higher speed, there is a decrease in total flow rate (see Fig. 5b). So the leakage flow rate in case of gradual expansion also dropped with increasing rotational speed giving rise to the efficiency. There is a drop in efficiency for \(N \leq 22\) rpm including the BEP. The increased flow rate, therefore the increased leakage flow rate and the hydraulic losses at the wheel control volume, describes this drop in efficiency. The BEP in both cases is attained at 14 rpm. The performance at BEP is summarized in Table 3.

<table>
<thead>
<tr>
<th>Transition</th>
<th>(\eta) (%)</th>
<th>(P_{\text{out}}) (W)</th>
<th>(N) (rpm)</th>
<th>(\tau) (Nm)</th>
<th>(H) (m)</th>
<th>(Q) (l/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gradual</td>
<td>59.8 ± 2.28</td>
<td>27.47 ± 1.13</td>
<td>14</td>
<td>18.74</td>
<td>0.3766</td>
<td>12.42</td>
</tr>
<tr>
<td>Sudden</td>
<td>62.14 ± 2.93</td>
<td>27.74 ± 1.07</td>
<td>14</td>
<td>18.92</td>
<td>0.3770</td>
<td>12.06</td>
</tr>
</tbody>
</table>

The relative change in performance between the gradual and sudden transition is calculated using Eq. 11 and plotted in Fig. 5d. As described earlier, the flow rate is positive until 24 rpm speed with maximum of 20% increase at \(N = 0\) rpm. Beyond \(N = 24\) rpm, it remains in the negative quadrant and reaches up to \(-2.6\)% due to these changes in flow rate, the head changes similarly and stays negative for \(N \leq 20\) rpm and the rest positive with a maximum value of 0.9%.

Change in power output is negative until 28 rpm speed. For the rest of the operating range, it stays in the positive quadrant and shows an exponential gain with increasing speed. Efficiency change is negative for \(N \leq 26\) rpm with the drop of \(-2.31\)% at BEP. At higher speeds, change in efficiency is positive and reaches of up to \(+1.6\)%.

The performance of the wheel with sudden and gradual transition profiles is compared at two points outside the BEP. These two points refer to the \(N = 8\) rpm and \(N = 30\) rpm speed which are below and above the BEP speed respectively. These points are chosen visually on the performance curve. The relative change in variables between the different settings are summarized in Table 4. At \(N = 8\) rpm, increase in flow rate in gradual transition is suggestive of the decrease in efficiency. The drop in head due to the increased flow rate following Eq. 1 and probable hydraulic losses describes the negative change in power output. However, at \(N = 30\) rpm change in power output and efficiency both are positive with a small change of \(+0.18\)% and \(+0.39\)% respectively. The change in head is positive due to decreased flow rate. The negative change in flow rate means proportionally reduced leakage flow at this operating point (Eq. 7). Since the change in power output in both of these operating points is very small, there is no significant change in hydraulic effects due to the change in transition designs. The drop in efficiency at \(N = 8\) rpm is however noticeable and is predominantly contributed by the increased amount of leakage flow. At the same time, uncertainty bounds are large at lower speed. Therefore, it is not clear whether the performance really changed between these settings.

The test with different transition profiles showed that efficiency doesn’t necessarily improve with the gradual transition profile. The hydraulic losses are lower than in the case of sudden transition profile, which means more hydraulic power input is transferred to the wheel. This became apparent up to 24 rpm speed where
higher flow rate is required in gradual transition for the same rotational speed of the wheel inferring increased leakage losses and consequently drop in efficiency of the wheel. As the rotational speed increased, the leakage flow became smaller and the head slightly increased in gradual transition than in sudden transition profiles contributing to the increase in power output and efficiency. However, the performance change is insignificant at higher speed and lies within the defined uncertainty bounds.
Table 4: Performance comparison between sudden and gradual transition shapes at two different operating points

<table>
<thead>
<tr>
<th>Operating point</th>
<th>$H$</th>
<th>$Q$</th>
<th>$P_{\text{out}}$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 8 \text{ rpm}$</td>
<td>$-0.13%$</td>
<td>$+6.49%$</td>
<td>$-1.18%$</td>
<td>$-4.05%$</td>
</tr>
<tr>
<td>$N = 30 \text{ rpm}$</td>
<td>$+0.10%$</td>
<td>$-0.76%$</td>
<td>$+0.18%$</td>
<td>$+0.39%$</td>
</tr>
</tbody>
</table>

4. Conclusion

The Dethridge water wheel is a potential device for pico-hydropower for decentralised applications. This wheel could particularly be of interest for its use in rural decentralised areas for simple applications like lighting and battery charging facilities. A 1.2 m diameter wheel would produce about 350 W power at around 60% of efficiency, which could be considered an ample amount for simple applications.

The performance of the Dethridge water wheel, among others, depends on the geometry of the channel. Channel width is an important parameter for the installation of the Dethridge water wheel. The results show that on channels two times wider than the wheel, performance of the wheel improves while performance severely drops in channel width that is equal to the width of the wheel. That means, to install the wheel in an existing channel the wheel needs to be scaled according to the channel dimensions. The gradual transition shape should have performed better than the sudden transition considering the reduced amount of transition losses. However, the difference in performance between these two cases remained within the defined measurement uncertainties. As such, there is no requirement of a special flow guiding structure for the better performance of the Dethridge water wheel indicating simplicity in the civil construction works.

Acknowledgements

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References


**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>blade angle [$^\circ$]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>shroud angle [$^\circ$]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>efficiency [%]</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular velocity [rad/s]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>fluid density [kg/m$^3$]</td>
</tr>
<tr>
<td>$\tau$</td>
<td>torque [Nm]</td>
</tr>
<tr>
<td>$A$</td>
<td>flow area [m$^2$]</td>
</tr>
<tr>
<td>$B$</td>
<td>channel width [m]</td>
</tr>
<tr>
<td>$b$</td>
<td>wheel width [m]</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity [m/s$^2$]</td>
</tr>
<tr>
<td>$H$</td>
<td>total head [m]</td>
</tr>
<tr>
<td>$h$</td>
<td>flow depth [m]</td>
</tr>
<tr>
<td>$L$</td>
<td>transition length [m]</td>
</tr>
<tr>
<td>$l$</td>
<td>blade length [m]</td>
</tr>
<tr>
<td>$N$</td>
<td>rotational speed [rpm]</td>
</tr>
<tr>
<td>$P$</td>
<td>power output [W]</td>
</tr>
<tr>
<td>$Q$</td>
<td>flow rate [m$^3$/s]</td>
</tr>
<tr>
<td>$R$</td>
<td>wheel radius [m]</td>
</tr>
<tr>
<td>$r$</td>
<td>hub radius [m]</td>
</tr>
<tr>
<td>$V$</td>
<td>volume [m$^3$]</td>
</tr>
<tr>
<td>$v$</td>
<td>mean flow velocity [m/s]</td>
</tr>
<tr>
<td>$W_s$</td>
<td>work done on the wheel shaft [J]</td>
</tr>
</tbody>
</table>
Highlights

1. Dethridge wheel is a suitable device for very low head applications.

2. Channel geometry plays important role on the wheel performance.

3. Two times wider channel than wheel is required for better performance of wheel.

4. Gradual transition shapes have insignificant impact on wheel performance.