E2 decay strength of the M1 scissors mode of $^{156}$Gd and its first excited rotational state


1 Institut für Kernphysik, TU Darmstadt, Schlossgartenstr. 9, D-64289 Darmstadt, Germany
2 Department of Physics, Duke University and Triangle Universities Nuclear Laboratory, Durham, North Carolina 27708-0308, USA
3 Institut für Kernphysik, Universität zu Köln, Zülpicher Str. 77, D-50937 Köln, Germany
4 FF Mat and Institute EMMI and Research Division, GSI Helmholtzzentrum für Schwerionenforschung, Planckstr. 1, D-64291 Darmstadt, Germany
5 Frankfurt Institute for Advanced Studies FIAS, Ruth-Moufang-Str. 1, D-60438 Frankfurt am Main, Germany
6 School of Engineering and Computing, University of the West of Scotland, Paisley PA1 2BE, United Kingdom
7 SUPA, Scottish Universities Physics Alliance, Glasgow, G12 8QQ, United Kingdom

(Dated: April 20, 2017)

The E2/M1 multipole mixing ratio $\delta_{1\rightarrow2}$ of the $1^+_{\text{sc}} \rightarrow 2^+_{1}$ $\gamma$-ray decay in $^{156}$Gd and hence the isovector E2 transition rate of the scissors mode of a well-deformed rotational nucleus has been measured for the first time. It has been obtained from the angular distribution of an artificial quasi-monochromatic linearly polarized $\gamma$-ray beam of energy 3.07(6) MeV scattered inelastically off an isotopically highly-enriched $^{156}$Gd target. The data yield first direct support for the deformation dependence of effective proton and neutron quadrupole boson charges in the framework of algebraic nuclear models. First evidence for a low-lying $J^\pi = 2^+$ member of the rotational band of states on top of the $1^+$ band head is obtained, too, indicating a significant signature splitting in the $K = 1$ scissors mode rotational band.

Introduction. – Orbital out-of-phase oscillations of a coupled two-component many-body quantum system are generally called Scissors Modes (ScMs). A ScM has been discovered in deformed atomic nuclei [1]. It has later been identified in Bose-Einstein condensed gases [2, 3] and is expected to occur in Fermi gases [4], in metallic clusters [5–7], and in deformed quantum dots [8]. ScMs are interesting quantum modes because their properties are sensitive to the restoring forces between the many-body subsystems. They inevitably break spherical symmetry and hence lead to a sequence of quantum states of the many-body system that form a rotational band.

The isovector low-lying $J^\pi_K = 1^+$ ScM of deformed even-even nuclei is the most prominent example for a ScM. Its occurrence has been conjectured in 1978 by Lo Iudice and Palumbo [9] in the framework of the semi-classical two-rotor model (TRM) of coupled quadrupole-deformed proton and neutron subsystems. In the framework of the algebraic interacting boson model (IBM-2) [10] it was explicitly considered as a valence-shell mode by Iachello [11], who identified it as one example of an entire class [12] of nuclear valence-shell excitations with non-trivial symmetry properties with respect to the two coupled subsystems. Within the IBM-2 the proton-neutron symmetry of a wave function is quantified by the F-spin quantum number $F = F_{\text{max}} - 1$, where $F_{\text{max}}$ is given by half of the number of proton and neutron bosons $N = N_p + N_n$. We address states with $F = F_{\text{max}} - 1$ in this context as “isovector valence-shell excitations”.

While the nuclear ScM occurs due to the quadrupole deformation of the proton and neutron subsystems, its signature is the electromagnetic coupling to the ground-state band via strong magnetic dipole (M1) transitions caused by the predominant isovector character of its decay transitions to low-energy nuclear states with proton-neutron symmetry. Indeed, the ScM has been discovered [1] in the quadrupole-deformed nucleus $^{156}$Gd in inelastic electron-scattering experiments at Darmstadt. It manifested itself as an accumulation of $M1$ excitation strength concentrated in a few $J^\pi = 1^+$ states at excitation energies around 3 MeV. The ScM of deformed nuclei has been studied extensively in inelastic electron-scattering $(e,e')$, photon scattering $(\gamma,\gamma')$, and neutron-scattering $(n,n'\gamma)$ experiments [13–14] and Refs. therein. The observed correlation of the total $M1$ strength of the ScM to the size of the nuclear quadrupole-deformation parameter [15] [18] has proven the quadrupole-collective nature of the nuclear ScM. Despite of its quadrupole-collective origin, the electric quadrupole-decay (E2) properties of the ScM are, however, still unknown. Consequently, the predicted [19] deformation dependence of effective quadrupole charges in the IBM-2 has remained an open question. The ScM is expected to form a rotational band of states with spin and parity quantum numbers $J^\pi = 1^+, 2^+, 3^+$, etc. Evidence for an E2 excitation at an excitation energy about
21(5) keV above the dominant $1^+$ state of the ScM of $^{156}$Gd has been reported from $(e,e')$ experiments [20]. However, the data did not allow for establishing the corresponding $2^+$ state as a member of the rotational band built on top of the $1^+$ band head of the ScM.

It is the purpose of this Letter to present first data on the $E2/M1$ multipole-mixing ratio of a $\gamma$-ray transition between the ScM and the ground-state rotational band of the nucleus $^{156}$Gd. It represents the first measurement of an $F$-vector $E2$ transition in axially deformed nuclei. The data yield a finite difference of effective boson quadrupole-charges for proton and neutron bosons in the IBM-2 of a deformed nucleus fitted locally to sensitive $F$-scalar and $F$-vector transition rates in a single rotational nucleus. Moreover, the size of the measured $F$-vector $E2$ decay matrix element enables us to estimate the $E2$ excitation strength of the $2^+_1$ state of the ScM rotational band from an Alaga-rule constraint. The data indicate the existence of this state consistent with the estimated $E2$ excitation strength and with the previous $(e,e')$ data. Combined, the data hint at a significant signature splitting in the $K = 1$ rotational band of the ScM.

**Experiment and results.** Nuclear resonance fluorescence (NRF) experiments [13, 21] with linearly-polarized quasimonoenergetic $\gamma$-ray beams [22] have been performed at the High-Intensity $\gamma$-Ray Source (HI$\gamma$S) [23] at Duke University, Durham, NC. The photon beams were scattered off a Gd$_2$O$_3$ target which contained 10 g of Gadolinium with an enrichment of 93.79(3) % in $^{156}$Gd. The target was mounted in the $\gamma$-3 setup [24], which included four high-purity Germanium (HPGe) detectors at a polar angle of $\vartheta = 135^\circ$ with respect to the incoming beam and at azimuthal angles $\varphi$ of 0°, 90°, 180°, and 270° with respect to the horizontal polarization plane. The linear polarization of the incident photons causes an anisotropic azimuthal distribution of the scattered photons, which is sensitive to the $E2/M1$ mixing ratio of the $1^+_sc \to 2^+_sc$ transition [25, 26]. Its angular distribution function is given by

$$W(\vartheta, \varphi; \delta) = 1 + \frac{3}{40} \left( \frac{1 + 6\sqrt{3} \delta + 5\delta^2}{1 + \delta^2} \right) \cos^2 \vartheta + (1 - \cos^2 \vartheta) \cos 2\varphi - 1/3$$

with the $E2/M1$ multipole-mixing ratio

$$\delta_{1\to2} = \frac{\sqrt{3} E_{\gamma} \langle 2^+_2 \parallel \hat{T}(E2) \parallel 1^+_1 \rangle}{10 \hbar c \langle 2^+_2 \parallel \hat{T}(M1) \parallel 1^+_1 \rangle}$$

in the phase convention of Krane et al. [27]. The quantities $\hat{T}(E2)$ and $\hat{T}(M1)$ denote the electric quadrupole and magnetic dipole transition operators.

**FIG. 1.** (color online) Gamma-ray spectra from the $^{156}$Gd($\vec{\gamma},\gamma'$) reaction taken at the HI$\gamma$S facility [23]. Detectors were placed at a polar angle of $\vartheta = 135^\circ$ and azimuthally in the horizontal polarization plane, a), of the incident $\gamma$-ray beam and perpendicular to it, b). The $1^+_sc \to 0^+_1$ $M1$ transition with 3.070 MeV transition energy and the mixed $E2/M1$ transition to the $2^+_1$ state at 2.981 MeV dominate the spectra. Other fragments of the ScM are indicated by arrows in panel a). The energy profile of the $\gamma$-ray beam is indicated by the dashed Gaussian curve in panel b). The data points were obtained from the relative luminosity determined from the known cross sections [28] for the corresponding $0^+_1 \to 1^+ \to 0^+_1$ photon scattering cascades.

**TABLE I.** Comparison of measured relative intensities $I_{rel}$ of transitions from the strongest fragment of the ScM to values from Ref. [28]. In addition to the $1^+_sc \to 0^+_1$, $2^+_1$ transitions, recently discovered [22] decay paths of the $1^+_sc$ state at 3.970 MeV to other low-lying states are considered. The intensities are normalized to 100 for the ground-state transitions. Furthermore, the determined multipole-mixing ratio for the $1^+_sc \to 2^+_sc$ transition is given.

<table>
<thead>
<tr>
<th>Transition $E_{\gamma}$ (keV)</th>
<th>$\delta_{1\to2}$</th>
<th>$I_{rel}$ (%)</th>
<th>$I_{rel}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^+_sc \to 0^+_1$</td>
<td>3070</td>
<td>100.0(3)</td>
<td>100(27)</td>
</tr>
<tr>
<td>$1^+_sc \to 2^+_1$</td>
<td>2981 - 0.07(1)<em>{stat}(2)</em>{syst}</td>
<td>48(1)</td>
<td>59(6)</td>
</tr>
<tr>
<td>$1^+_sc \to 0^+_2$</td>
<td>2020</td>
<td>0.35(8)</td>
<td></td>
</tr>
<tr>
<td>$1^+_sc \to 2^+_2$</td>
<td>1941</td>
<td>0.48(10)$^a$</td>
<td>0.46(9)$^b$</td>
</tr>
<tr>
<td>$1^+_sc \to 0^+_3$</td>
<td>1902</td>
<td>0.36(9)</td>
<td></td>
</tr>
</tbody>
</table>

$^a$assuming pure $M1$ character

$^b$assuming pure $E2$ character

Figure 1 shows the ($\gamma, \gamma'$) spectra of $^{156}$Gd measured in the polarization plane a) and perpendicular to it b). The energy profile of the beam with a width of about 3.5 % of the centroid energy 3.070 MeV is indicated by a dashed Gaussian in the lower plot. Prominently observed – be-
The multipole mixing ratio can be shown to be incompatible with the fact that the $\gamma$ + state at 3.070 MeV is the strongest $M1$ excitation of $^{156}$Gd [28]. Hence, application of the Alaga rule is well justified for this predominantly axially deformed nucleus $^{156}$Gd, in particular for transitions whose strengths dominate in the given energy interval. The $^{156}$Gd $\gamma$-ray beam, respectively. The observed relative transition strengths between arbitrary states of two rotational bands of an axially-deformed rotor differing by $\Delta K = K_f - K_i$ in their intrinsic angular-momentum projection quantum number. The reduced relative decay-intensity ratio $R_{\exp} = [I_{rel,1^+ \rightarrow 2^+}^{156\text{Gd}}/I_{rel,1^+ \rightarrow 2^+}^{156\text{Gd}}]^{1/2}$ is consistent within two standard deviations with the value $R(\Delta K = 1) = 0.5$ from the Alaga rule for pure dipole transitions from the ScM with intrinsic projection $K = 1$ to the $K = 0$ ground-state band. This already indicates that the $1^+_{sc} \rightarrow 2^+_{1}$ transition is dominated by the $M1$ component and a possible $E2/M1$ multipole mixing ratio must be close to zero, i.e., $\delta_1^{2\rightarrow1} \ll 1$.

In order to obtain information on the size of the $E2$ decay matrix element from the ScM to the ground-state band, we have analyzed the $E2$ contribution to the $1^+_{sc} \rightarrow 2^+_{1}$ transitions. The ratio $N_\parallel/N_\perp$ is sensitive to the multipole mixing ratio $\delta_{1 \rightarrow 2}$. Here, $N_\parallel$ and $N_\perp$ are the $\gamma$-ray intensities registered in and perpendicular to the polarization plane of the incident $\gamma$-ray beam, respectively. The observed ratio is compared to the respective ratio of angular distributions $W(135^\circ,0^\circ; \delta_1 \rightarrow 2\gamma)/W(135^\circ,90^\circ; \delta_1 \rightarrow 2\gamma)$ from Eq. (1) for the $0^+_1 \gamma$ $\rightarrow 1^+_{sc,1} \rightarrow 2^+_1$ sequence, integrated over the opening angles of the individual detectors. The result for the $1^+_{sc,1} \rightarrow 2^+_1$ decay transition at 2.981 MeV (cf. Fig. 1) is shown in Fig. 2. It features two solutions: one close to zero, the other corresponding to dominant $E2$ character. The first solution, $\delta_{1 \rightarrow 2} = -0.07(1)_{\text{stat}}(2)_{\text{syst}}$, is the only one consistent with both, the data for the azimuthal angular distribution and with the Alaga rule. Evidence for possible $K$ mixing has been shown to be small (cf. e.g. Ref. [31] and Fig. 9 in Ref. [14]). In the case of two-state mixing the alternative value $\delta_{1 \rightarrow 2} = -2.69(19)$
formation on the strength of an $F$-vector $E2$ transition in an axially-deformed nucleus.

In the IBM-2 an $F$-vector $E2$ transition strength is proportional to the square of the difference of the effective boson quadrupole charges for proton and neutron bosons $(e_\pi - e_\nu)^2$. Using the $F$-spin limit of the SU(3) dynamical symmetry of the IBM-2 and considering that the ScM fragment at 3.070 MeV carries about 1/3 of the boson quadrupole charges for proton and neutron bosons proportional to the square of the difference of the effective boson quadrupole charges directly from the Fermi-liquid approach to boson quadrupole charges obtained from previous approaches. These fitted effective charges to $B(E2; 2^+ \rightarrow 0^+)$ values from a chain of isotopes assuming that their structure does not differ, and that the model qualitatively reproduces the evolution of the data as a function of boson number. Instead, in the present work they are locally determined from an $F$-vector $E2$ transition discussed above and from the transition strength of 189(3) W.u. for the $F$-scalar $E2$ transition from the $2^+_1$ state to the ground state of the very same nucleus, $^{156}$Gd. The corresponding reduction of $(e_\pi - e_\nu)^2$ by about one order of magnitude with respect to previous estimates, and a correspondingly small $E2$ excitation strength from the ground state, explains why the $2^+_2$ state has remained largely undetected in the past. The new data agree with early predictions for the deformation dependence of the effective $E2$ boson charges in the IBM-2.

The Alaga rule can be used for estimating the $M1$ and $E2$ transition rates from the expected states of the rotational bands, built on top of the fragments of the ScM, to the ground-state band from the measured $B(M1; 1^+_{sc}, K = 1 \rightarrow 0^+_{sc}, K = 0)$ and $B(E2; 1^+_{sc} → 2^+_1, K = 0)$ values. Applying Eq. 9 one obtains the estimates given in Table I for the $E2$ and $M1$ strengths from the first two $2^+_2, 3^+_2$ members of the rotational band expected to be built on top of the $1^+_{sc,1}$ state at 3.070 MeV.

From the estimated transition strengths one must expect that the $2^+_2$ state on top of the $1^+_{sc,1}$ state would decay to 99% to the $2^+_1$ state with an $E2/M1$ multipole mixing ratio of $\delta^2 \approx 0$. This expectation, together with the estimated $E2$ excitation strength of that $2^+_2$ state and the experimental luminosity curve (cf. Fig. 1), is consistent with the following experimental observation: A suitable peak at 3.000(1) MeV, observed in all detectors and interpreted as a signal for the $2^+_2$ state located 21(5) keV above the $1^+_{sc,1}$ state at 3.089 MeV, is shown in Figure 3. The experimental luminosity at 3.089 MeV together with the assumption that the Alaga estimates for the $2^+_2$ state from Table I are correct, would produce an excitation yield of about 500 counts in a peak at the transition energy of 3.000(1) MeV with a unique azimuthal intensity asymmetry of 0.176 for a $0^+ \rightarrow 2^+ \rightarrow 1^+ \rightarrow 2^+_1$ sequence. The measured asymmetry of the peak at 3.000(1) MeV amounts to

$$\frac{N_0 - N_\parallel}{N_\parallel + N_\perp} = 0.20(11),$$

excluding all reasonable alternatives to the spin sequence indicated above.

Hence, our data provide evidence for a $2^+$ state located about 19(1) keV above the $1^+_{sc,1}$ state, with an $E2$ excitation strength that matches the $E2$ decay strength of the dominant $1^+$ ScM level. This observation further coincides within uncertainties with the $2^+$ state found in previous inelastic electron scattering experiments at 21(5) keV above the $1^+_{sc,1}$ state as the strongest $E2$ excitation in that entire energy range. If indeed the

<table>
<thead>
<tr>
<th>Observable</th>
<th>Experiment</th>
<th>Alaga estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(M1; 1^+<em>{sc,1} \rightarrow 0^+</em>{sc})$</td>
<td>0.451(39)</td>
<td>0.226(20)</td>
</tr>
<tr>
<td>$B(M1; 1^+<em>{sc,1} \rightarrow 2^+</em>{1})$</td>
<td>0.246(21)</td>
<td>0.74(6)</td>
</tr>
<tr>
<td>$B(M2; 2^+<em>{sc} \rightarrow 2^+</em>{1})$</td>
<td>0.42(4)</td>
<td>0.32(3)</td>
</tr>
<tr>
<td>$B(M1; 3^+<em>{sc} \rightarrow 2^+</em>{1})$</td>
<td>0.037(26)</td>
<td>0.018(10)</td>
</tr>
<tr>
<td>$B(M1; 3^+<em>{sc} \rightarrow 3^+</em>{1})$</td>
<td>0.005(4)</td>
<td>0.017(12)</td>
</tr>
<tr>
<td>$B(E2; 1^+<em>{sc,1} \rightarrow 2^+</em>{1})$</td>
<td>0.011(7)</td>
<td>0.026(19)</td>
</tr>
</tbody>
</table>

FIG. 3. (color online) Candidate for the $2^+ \rightarrow 2^+_1$ transition at 3.000(1) MeV observed in all four HPGe detectors. The peak contains about 640(90) counts. Its intensity and azimuthal asymmetry suggest its interpretation as the decay transition to the $2^+_1$ state of the $2^+_2$ state at an excitation energy of 3.089 MeV.
2\textsuperscript{+} state at 3.089(1) MeV excitation energy is the first rotational excitation of the 1\textsuperscript{+}\textsubscript{sc} state at 3.070 MeV, two cases have to be considered. The scissors mode either has a rotational moment of inertia which exceeds the rigid-body value by more than 50%, or the K = 1 ScM rotational band would exhibit a significant signature splitting [34] with a decoupling parameter\[\text{of} 0.34,\] assuming the moment of inertia of the scissors-mode state. Its identification, e.g., in (e, e') experiments, is therefore of great importance, also with respect to entanglement in the two-rotor model [35] or to energy shifts due to multi-state mixing and, hence, the formation of rotational bands, altogether.

Summary. – For the first time, we have measured the multipole mixing ratio of the decay transition from the scissors mode to the 2\textsuperscript{+} state in a deformed nucleus. We have determined the B(E2; 1\textsuperscript{+}\textsubscript{sc} → 2\textsuperscript{+}) value of 156\textsuperscript{Gd} and estimated the γ-decay behavior of the scissors-mode band from an Alaga rule constraint. The data provide direct evidence for a decrease of F-vector E2 boson charges within the IBM-2 as a function of ground-state deformation, and for the 2\textsuperscript{+}\textsubscript{sc} rotational state at 3.089(1) MeV excitation energy. This excitation energy poses a puzzle within the IBM-2 as a function of ground-state deformation. This alternative can only be verified by a future observation of the 3\textsuperscript{+}\textsubscript{sc} state. Its identification, e.g., in (e, e') experiments, is therefore of great importance, also with respect to entanglement in the two-rotor model [35] or to energy shifts due to multi-state mixing and, hence, the formation of rotational bands, altogether.

The authors thank the HI\textsuperscript{2} accelerator crew for providing perfect experimental conditions and N. Lo Iudice, R. Jolos, and V.O. Nesterenko for valuable discussions. This work was partly supported by the Deutsche Forschungsgemeinschaft under Grant Nos. SFB 634, SFB 1245, and ZI 510/7-1, partly by the BMBF under Grant Nos. SFB 634, Forschungsgemeinschaft under Grant Nos. SFB 634, and, hence, the formation of rotational bands, altogether.

The authors thank the HI\textsuperscript{2} accelerator crew for providing perfect experimental conditions and N. Lo Iudice, R. Jolos, and V.O. Nesterenko for valuable discussions. This work was partly supported by the Deutsche Forschungsgemeinschaft under Grant Nos. SFB 634, SFB 1245, and ZI 510/7-1, partly by the BMBF under Grant No. 05P15RDEN9, by the Alliance Program of the Helmholtz Association (HA216/EMMI) and by the U.S. DOE Grant No. DE-FG02-97ER41033.


\[\text{1} \text{defined for K = 1 according to the definition from Ref. [34] for K = 1/2.}\]