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A new flower pollination algorithm for equalization in synchronous DS/CDMA multiuser communication systems

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Abstract This work proposes a modified version of an emerging nature-inspired technique, named Flower Pollination Algorithm (FPA), for equalizing digital multiuser channels. This equalization involves two different tasks: 1) estimation of the channel impulse response, and 2) estimation of the users' transmitted symbols. The new algorithm is developed and applied in a Direct-Sequence / Code-Division Multiple-Access (DS/CDMA) multiuser communications system. Important issues such as robustness, convergence speed and population diversity control have been in deep investigated. A method based on the entropy of the flowers' fitness is proposed for in-service monitoring and adjusting population diversity. Numerical simulations analyze the performance, showing comparisons with well-known conventional multiuser detectors such as Matched Filter (MF), Minimum Mean Square Error Estimator (MMSEE) or several Bayesian schemes, as well as with other nature-inspired strategies. Numerical analysis shows that the proposed algorithm enables transmission at higher symbol rates under stronger fading and interference condi-

tions, constituting an attractive alternative to previous algorithms, both conventional and nature-inspired, whose performance is frequently sensible to near-far effects and multiple-access interference problems. Several statistical tests have also been conducted in order to prove the validity of results.

Keywords DS/CDMA · Nature-inspired algorithms · Flower pollination algorithm · Channel estimation · Symbol detection · Multiuser detection · Population diversity.

1 Introduction

Nowadays communication systems face up the problem of high transmission rates over limited bandwidth channels, with an increasing number of users. In such scenarios, efficient radio channel sharing techniques have deserved special attention in scientific community during last years. Direct Sequence / Code-Division Multiple-Access (DS/CDMA) is one of the most widely used and studied channel sharing methods. Currently, this technology is implemented in many real wireless communication systems, such as LTE, UMTS, CDMA2000, IS-95 and even in global positioning applications (US GPS, European Galileo and Russian GLONASS). Besides, CDMA is present in 5G systems in combination with other technologies as well as in cordless phones operating in the 900 MHz, 2.4 GHz and 5.8 GHz bands, in IEEE 802.11b 2.4 GHz wifi, and in radio controlled model automotive vehicles. In modern systems where security is a key design aspect, DS/CDMA has been proposed combined with both chaotic spreading sequences (Rahnama & Talebi, 2013) and with MIMO techniques (Mehrizi & Mahboobi, 2017) to decrease the

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level of interferences, improving, this way, other sharing schemes.

Due to its extensive application and use, DS/CDMA systems constitute a well-known and widely studied problem, thus allowing direct comparison to previously developed multiuser detectors.

Since current CDMA systems allow transmission at very high rates, intersymbol and multiaccess interferences (ISI and MAI) must be taken into account and controlled so that performance does not degrade (Proakis, 1998). These interferences, if not limited, will surely drop system's performance. To address this problem, conventional multiuser detectors (MUDs) make use of filters matched to the codeword of the user of interest. However, this scheme is only optimum when all received codewords are independent, which rarely occurs in real applications and performance notably decreases, especially if near-far effects are also present. In 1998, S. Verdú found that the joint extraction of the users transmitted sequences could mitigate this problem (Verdu, 1998). However, the optimum MUD relying on the maximum likelihood (ML) criterion has a complexity that is exponentially proportional to the number of system users, making it unfeasible in real scenarios. Therefore, many authors have paid attention to the development of suboptimal schemes that can be implemented on real systems. An important branch of methods are those based on meta-heuristics, specially, nature-inspired approaches. These have been successfully tested for up to 2,000 dimensions for large scale optimization (Agarwal & Mehta, 2014).

Among these nature-inspired approaches the following can be highlighted: Genetic Algorithms (GAs) were the first and simplest methods. Juntti et al. (1997) proposed a synchronous DS/CDMA MUD based on GAs. Its main drawback was the requirement of good estimates of the first transmitted symbols. A variant of the GA was proposed by Yen & Hanzo (2000), whose performance is close to the optimum thanks to a local search algorithm introduced before the GA. This idea of adding modules to the standard GA was also used by Ergun & Hacioglu (2000). In this case a multistage detector is integrated into the GA in order to speed up convergence. Later, Yen & Hanzo (2004) studied the asynchronous case taking into account the effect of the surrounding symbols of the other system users.

Other recent works include (Maradia et al., 2009), which focuses on selective Rayleigh fading channels estimated using a GA-based detector. The approach by Dong et al. (2004), which studies both bit-error-rate (BER) and near-far effect performances. Some other worth-mentioning recent approaches using GAs are (Yen & Hanzo, 2001; Shayesteh et al., 2003; Tan et al., 2010a,b;

Guntu Nooka & Rao, 2013; Khan et al., 2015; Huang et al., 2017).

Apart from GAs, several other nature-inspired methods have also been applied to multiuser detection. For instance: tabu search (TS) has been proposed in (Tan & Rasmussen, 2004; Driouch et al., 2010; Datta et al., 2010; Srinidhi et al., 2011). A relevant simulated annealing (SA)-based strategy was proposed by Tan & Rasmussen (2004). Hybrid methods combining SA with particle swarm optimization, genetic algorithms and a Hopfield neural network have been proposed in (Gao & Diao, 2009; Yao et al., 2011; Tan et al., 2013), respectively. In methods such as TS or SA, a single solution is obtained throughout search, therefore quality of solution highly depends on initial guesses.

The more advanced particle swarm optimization (PSO) (De Oliveira et al., 2006; Soo et al., 2007; Wang, 2014; Arani & Azmi, 2013; Kaur, 2016), cat swarm optimization (CSO) (Pradhan & Panda, 2012; Sohail et al., 2017; Balamurugan et al., 2018), the cuckoo search algorithm (CSA) (Agarwal & Mehta, 2014; Yang & Deb, 2014), ant-colony algorithms (ACO) (Hijazi & Nataraajan, 2004; Lain & Lai, 2007; Xu et al., 2007; Marinello Filho et al., 2012) or the Flower Pollination Algorithm (FPA) (Yang, 2012; Balasubramani & Marcus, 2014; Pant et al., 2017; Abdel-Baset et al., 2017a; Abdel-Basset & Shawky, 2019), among others. See (Larbi et al., 2014) for an interesting list of references using heuristic methods in CDMA communication problems.

Existing literature reveals that the main drawbacks in solving real-world problems found in these meta-heuristics are: computational complexity, slow convergence speed and suboptimal convergence to local minima, mainly due to the difficulty to maintain population diversity while iterations run, and the impossibility to detect and escape from these suboptimal solutions (Tan & Rasmussen, 2004; Agarwal & Mehta, 2014; Larbi et al., 2014; Arora & Anand, 2017; Sohail et al., 2017).

In this paper we focus on the development of a new nature-inspired algorithm, based on the standard Flower Pollination Algorithm (FPA) –initially proposed by Yang (2012)– and its application to channel estimation and symbol detection in DS/CDMA environments. Among the main advantages of FPA we find: easy implementation, shorter computation time, broad flexibility, robust performance, intuitive structures and guarantee of fast convergence (Yang, 2012; Abdel-Raouf et al., 2014; Dubey et al., 2015; Nigdeli et al., 2016; He et al., 2017; Abdel-Basset & Shawky, 2019). In fact, FPA has already been successfully used in complex applications such as: constrained global optimization (Abdel-Raouf et al., 2014), economic dispatch (Prathiba et al., 2014;

Dubey et al., 2015), assembly sequence optimization (Mishra & Deb, 2019), analysis of micro-CT scans (Kowalski et al., 2019), simulation of photovoltaic systems (Alam et al., 2015), EEG-based person identification (Rodrigues et al., 2016), knapsack problems (Abdel-Basset et al., 2018; Abdel-Basset & Zhou, 2018), the spherical traveling salesman problem (Zhou et al., 2019), ill-conditioned sets of equations (Abdel-Baset & Hezam, 2016), quadratic assignment (Abdel-Baset et al., 2017b), antenna positioning (Dahi et al., 2016), wireless sensor optimization (Sharawi et al., 2014), or sizing optimization of truss structures (Bekdaş et al., 2015), among others. Recently, FPA has also been applied in hybrid proposals with other nature-inspired algorithms, such as tabu search, simulated annealing or particle swarm optimization (Lenin et al., 2014; Abdel-Baset & Hezam, 2015; Hezam et al., 2016; Abdel-Baset & Hezam, 2016). A detailed review of FPA applications can be found in (Balasubramani & Marcus, 2014; Chiroma et al., 2015; Pant et al., 2017).

Our proposal introduces some modifications to the standard FPA in order to: i) improve search convergence by monitoring and controlling population diversity using the entropy of the population fitness, and 2) reduce its complexity by carefully adjusting the probability of each pollination type (local/ global) and considering different applications of these pollinations to certain parts of the potential solutions –see section 3.

To the best of our knowledge, no previous attempts to make use of FPA for joint fading estimation and symbol detection in communications problems have been published.

The rest of the paper is organized as follows: section 2 explains the joint channel estimation and symbol detection problem in a DS/CDMA communications system. Basic concepts and notation are here presented, along with the proposed fitness function to be used in the FPA. Section 3 develops the proposed FPA and shows how population diversity is controlled using the population fitness entropy. Next, section 4 shows the numerical results with emphasis to comparison with both conventional and nature-inspired multiuser detectors. Finally, conclusions and future research are outlined in section 5.

2 DS/CDMA multiuser communication channel model

This section describes the channel model used, where U binary symbol sequences are simultaneously transmitted over a shared radio channel. Each user makes use of a private normalized modulation signature from set $\{s_i(t)\}_{i=1}^U$. Channel response is considered to be

completely characterized by a set of flat-fading coefficients, and an additive white zero-mean Gaussian noise (AWGN). A perfect synchronization of all signals is assumed.

User i transmits an F -length sequence $d_i(n)$ of statistically-independent symbols that modulate codeword $s_i(t)$. This way, the i th user transmits the following signal

$$x_i(t) = \sum_{n=0}^{F-1} d_i(n)s_i(t - nT) \quad (1)$$

where $d_i(n)$ are the users' transmitted symbols and T is the symbol period. User signatures (or codewords) are obtained as

$$s_i(t) = \sum_{\ell=0}^{N-1} s_{i,\ell}\gamma(t - \ell T_c) \quad (2)$$

where $\mathbf{s}_i = (s_{i,0}, \dots, s_{i,N-1})^T$ stands for the signature of user i , $T_c = T/N$ is the chip period (N is known as *processing gain*) and $\gamma(t)$ denotes a chip waveform with normalized energy. As a consequence, the spectrum width is spread by a factor N , de-sensitizing, this way, the original narrowband signal to some potential channel degradation and interference (Proakis, 1998). As we are considering a synchronous system, the signal at the receiver input is

$$r(t) = \sum_{i=1}^U r_i(t) + g(t) \quad 0 \leq t \leq T_F \quad (3)$$

where $g(t)$ denotes an AWGN, which is not correlated with symbols $d_i(n)$, T_F represents the frame duration and $r_i(t)$ is

$$r_i(t) = \sqrt{E_i} \sum_{n=0}^{F-1} a_i(n)d_i(n)s_i(t - nT) \quad (4)$$

E_i denotes the bit energy of the i th user, and $a_i(n)$ is the flat-fading coefficient of the i th user. In this work, a non-stationary channel is considered, with a time variation model defined in (Yen & Hanzo, 2001), where fading coefficients $a_i(n)$ change with time as a function of a Doppler frequency, f_d , as

$$a_i(n+1) = \alpha \cdot a_i(n) + \nu \quad (5)$$

with $\alpha = e^{-2\pi f_d T}$ and ν representing an AWGN component.

The main aim of our algorithm –the joint channel estimation and symbol detection task– can be seen in Eq. (4), where both the users' data symbols as well as the flat-fading coefficients must be estimated.

The first step in the receiver consists of a bank of filters matched to the users' codewords, just after sampling the received signal at $1/T$ rate –see Fig. 1.

In order to estimate the vector of transmitted symbols $\mathbf{d}(n) = [d_1(n), \dots, d_U(n)]^T$, we have followed ideas

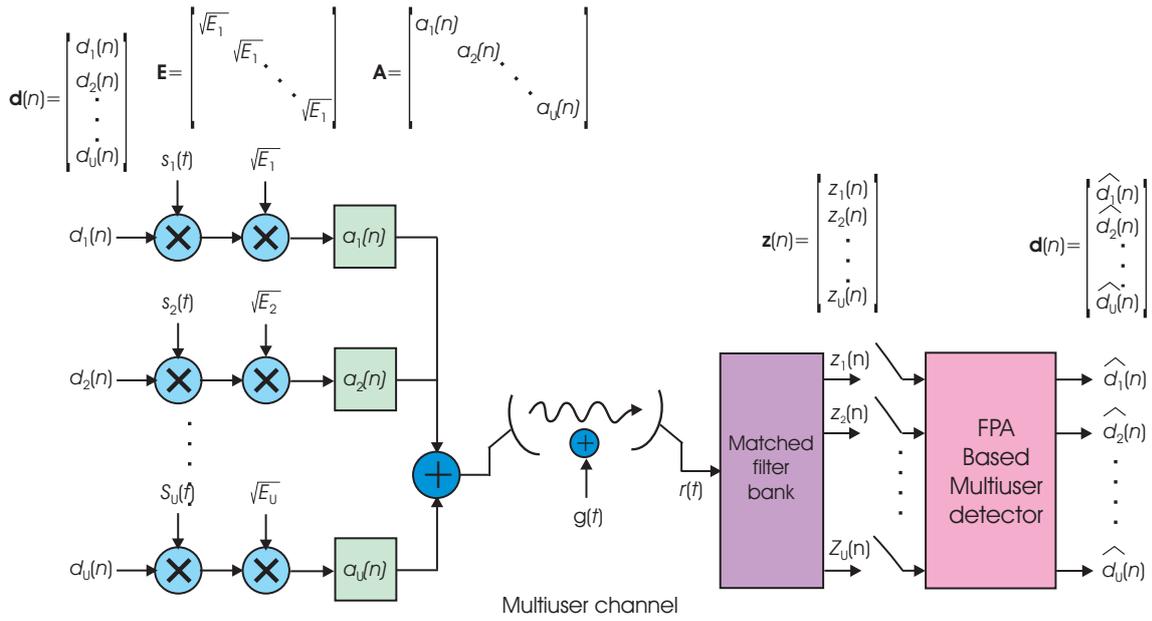


Fig. 1 Multiuser DS/CDMA system model with U active users. E_i : bit energy of user i , $d_i(n)$: symbol sequence transmitted by user i , $s_i(t)$: codeword of user i , $\hat{d}_i(n)$: estimate of n th symbol transmitted by user i . a_i : channel flat-fading coefficient of user i .

in (Yen & Hanzo, 2001), where estimation is presented as a maximization problem, and the output of the matched filters' bank, $\mathbf{z}(n)$, is obtained as

$$\mathbf{z}(n) = [z_1(n), \dots, z_U(n)]^T = \mathbf{R}\mathbf{A}(n)\mathbf{E}\mathbf{d}(n) + \mathbf{g}(n) \quad (6)$$

with \mathbf{R} being the $U \times U$ cross-correlation matrix of users' codewords, $\mathbf{A}(n) = \text{diag}(a_1(n), \dots, a_U(n))$, $\mathbf{E} = \text{diag}(\sqrt{E_1}, \dots, \sqrt{E_U})$, $\mathbf{d}(n) = [d_1(n), \dots, d_U(n)]^T$ and $\mathbf{g} = [g_1(n), \dots, g_U(n)]^T$.

Considering vector \mathbf{z} , it can be demonstrated that the log-likelihood conditional pdf given both the fading coefficients' matrix $\mathbf{A}(n)$ and the users' data vector $\mathbf{d}(n)$, is (Fawer & Aazhang, 1995)

$$\mathcal{L}(\mathbf{A}(n), \mathbf{d}(n)) = 2\Re \{ \mathbf{d}(n)^T \mathbf{E}[\mathbf{A}(n)]^* \mathbf{z}(n) \} - \mathbf{d}(n)^T \mathbf{E}\mathbf{A}(n)\mathbf{R}[\mathbf{A}(n)]^* \mathbf{E}\mathbf{d}(n) \quad (7)$$

with " \Re " and " $*$ " representing the real part of a complex magnitude, and the complex conjugate operator, respectively. Hence, estimates of the matrix with the fading coefficients and the vector of data symbols are obtained as

$$(\hat{\mathbf{A}}(n), \hat{\mathbf{d}}(n)) = \arg \max_{\mathbf{A}(n), \mathbf{d}(n)} \{ \mathcal{L}(\mathbf{A}(n), \mathbf{d}(n)) \} \quad (8)$$

On the other hand, time variation of fading coefficients is supposed to be slow enough so as to consider fading response to be constant within each symbol period. Besides, fading from different users are considered independent.

Notice that this receiver works at symbol rate $1/T$. However, it is also possible to develop an algorithm

operating at chip rate, $1/T_c = N/T$. A brief introduction to this approach is developed in A.

3 Flower pollination algorithm (FPA) description

3.1 Basic concepts and standard FPA

Standard FPA (Yang, 2012) was based on modeling the natural pollination process in nature. This process is necessary for reproduction in those plants with flowers (angiosperms), which are about 80% of all plant species. The pollination process transfers pollen from some flowers to others, with the aim of reproduction. Two types of pollination can be considered: biotic (pollen is transferred by insects or animals) and abiotic (the wind or the rain carry out the process of pollination).

Two different types of pollination can be considered: local (or *auto-pollination*) and global (or *cross pollination*). In the later, pollen is transferred from the flower of another plant; while, in local pollination, pollen comes from the same flower, or flowers, belonging to the same plant, usually without the action of a pollinator.

Global pollination is normally biotic and pollen travels long distances due to the action of pollinators such as birds, insects, bees or bats, whose travels can be modeled using a Lévy distribution (Pavlyukevich, 2007).

According to (Yang, 2012), the basic process of pollination can be idealized considering that: (i) biotic cross pollination is *global* pollination, (ii) pollinators use Lévy flights to carry pollen, (iii) auto-pollination (abiotic) is

will compose the population, resulting in a very low diversity. In this paper, we propose to use a probability of change that depends on the Shannon entropy of the population fitness, which can be computed as

$$\mathcal{H}(\mathcal{P}[k]) = - \sum_{i=1}^{NumFl} \mathcal{L}_i^*[k] \log \mathcal{L}_i^*[k] \quad (16)$$

with $\mathcal{L}_i^*[k]$ being the normalized fitness of flower \mathbf{x}_i , i.e. $\mathcal{L}_i^*[k] = \mathcal{L}_i[k] / (\sum_{j=1}^{NumFl} \mathcal{L}_j[k])$. Fitness $\mathcal{L}_i[k]$ is calculated before normalization using Eq. (7). This way, when fitness values are all of them very similar, with small dispersion, $\mathcal{H}(\mathcal{P}[k])$ becomes high, and P_c is increased in order to boost global pollination through Eq. (11). Thus, diversification is performed. On the other hand, a small value of $\mathcal{H}(\mathcal{P}[t])$ reveals a high diversity within population, and P_c is decreased so as to promote local pollination through Eq. (15).

At this point, it is important to notice that entropy will be high both during first iterations (fitness values are low since they are initial estimates randomly generated) and last iterations (if convergence has been achieved most of the flowers will encode similar solution estimates representing good estimates of the solution with a high associated fitness value). This situation is easy to manage since we know if the algorithm is in the first or last iterations.

Another particularity is that pollination operators given by Eqs. (11) and (15) are now applied individually to each part of flower $\mathbf{x}_i[t]$ –see Eq. (9). Global pollination will affect the right-most part of $\mathbf{x}_i[t]$ (regarding flat fading coefficients) with higher probability during the first iterations. After an initial period of symbols, this part will be affected almost only by local pollination. On the other hand, the left side of $\mathbf{x}_i[t]$ (encoding users' transmitted data) will be affected following the standard FPA principles. The reason for this is that channel coefficients vary slowly with time and can be considered constant along several symbol periods. Once they are estimated, only a fine-tuning of them is necessary. Transmitted symbols are considered independent between different symbol periods, so FPA must be initialized with randomly generated symbol estimates, for every transmitted symbol.

In order to perform this adjustment, flower coding is divided into two zones of different pollination. The part of the fading coefficients ($\mathbf{a}(n)$) do not need to be re-initialized when the algorithm is run in each symbol period. On the contrary, the part encoding transmitted symbols ($\mathbf{d}(n)$) is randomly re-initialized at the beginning of each symbol period –see structure of $\mathbf{x}_i[t]$ in Eq. (9).

According to this, and taking into account that $\mathbf{x}_i[t] = [\mathbf{d}(n)|\mathbf{a}(n)]$, local pollination is performed with a lower

γ_a for the flat-fading coefficients part, and with γ_b ($\gamma_b > \gamma_a$) for the $\mathbf{d}(n)$ part of $\mathbf{x}_i[t]$. Specifically, $\gamma_b(0) = 0.1$ and remains constant with time, while $\gamma_a(0) = 0.05$ and decreases 10% every $MaxIter/4$.

The proposed FPA can be summarized using the following steps:

1. The iteration counter is initialized to 0, and an initial population of $NumFl$ flowers (\mathbf{x}) is established with randomly generated values. The best solution \mathbf{b}^* in this initial population is found (the one with the lowest fitness value). Probability of change $P_c(0)$ is set to 0.35.
2. A random value $r \in [0, 1]$ is calculated. If $r < P_c$ then go to step 3 (global pollination), otherwise, go to step 4 (local pollination).
3. Jump vector \mathbf{L} is estimated using the Lévy distribution given by Eq. (12). Then, global pollination is applied according to:

$$\mathbf{d}_i[t+1] = \mathbf{d}_i[t] + \gamma_d \mathbf{L}(\mathbf{d}^* - \mathbf{d}_i[t]) \quad (17)$$

$$\mathbf{a}_i[t+1] = \mathbf{a}_i[t] + \gamma_a \mathbf{L}(\mathbf{a}^* - \mathbf{a}_i[t]) \quad (18)$$

with $\gamma_d > \gamma_a$, and \mathbf{d}^* and \mathbf{a}^* representing the corresponding part in \mathbf{b}^* (current best solution), i.e. $\mathbf{b}^* = [\mathbf{d}^*|\mathbf{a}^*]$.

4. ϵ is calculated using a uniform distribution in $[0,1]$. Flowers j and k are randomly selected from current population ($1 \leq j, k \leq NumFl, j \neq k$). Local pollination is applied using Eq. (15).
5. Fitness value for each flower is calculated using Eq. (7). If the new fitness value of each flower is better than the previous one, then flower \mathbf{x} is updated. If not, the solution from previous iteration is kept.
6. Entropy of population fitness is evaluated using Eq. (16) and P_c is accordingly modified:
 - $\mathcal{H}(\mathcal{P}[t]) \uparrow \Rightarrow$ low dispersion $\Rightarrow \uparrow P_c \Rightarrow \uparrow$ exploration.
 - $\mathcal{H}(\mathcal{P}[t]) \downarrow \Rightarrow$ large dispersion $\Rightarrow \downarrow P_c \Rightarrow \uparrow$ exploitation.

Modifications of P_c are smaller as iteration t increases and are only applied every $MaxIter/10$ iterations.

7. Once every flower has been analyzed and pollination has been applied, the gamete with the best fitness is selected and assigned to $\mathbf{b}^* = [\mathbf{d}^*|\mathbf{a}^*]$.
8. Algorithm finishes if the preset maximum number of iterations ($MaxIter$) is reached. If not, go to step 2.

Fig. 2 shows a flowchart corresponding to the proposed FPA.

When the algorithm finishes, the flower with the highest fitness will represent the problem solution, i.e.

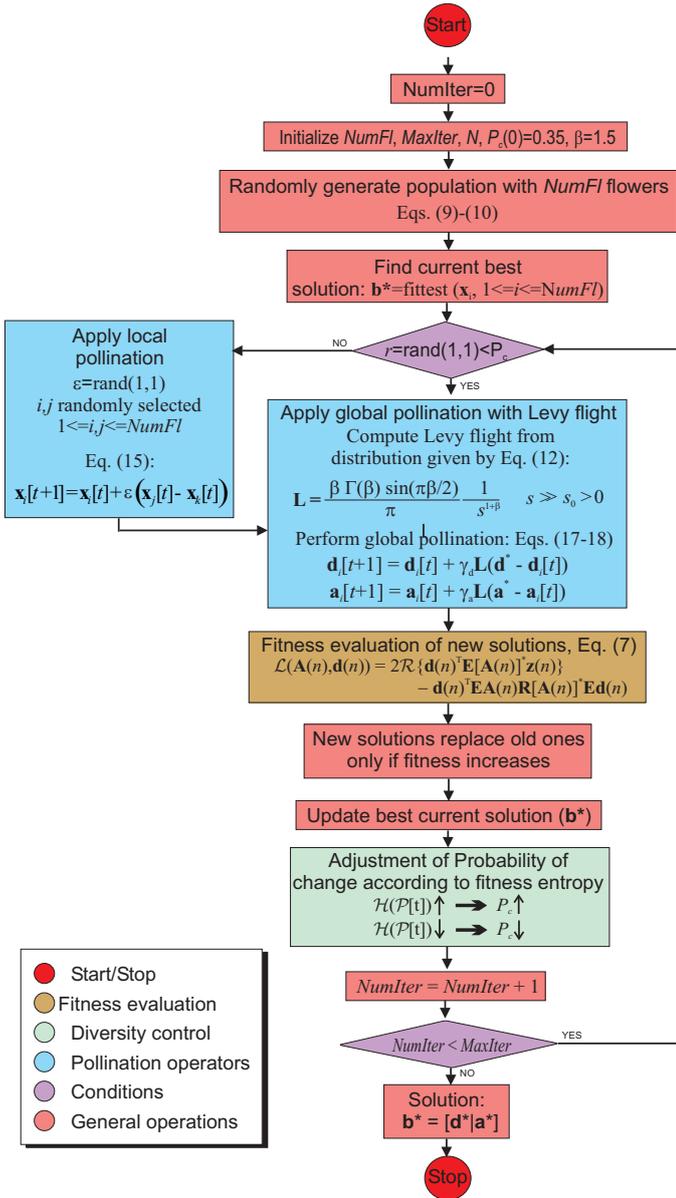


Fig. 2 Flowchart of the proposed FPA (flower pollination algorithm).

the estimate of both the symbols' vector \mathbf{d} and the matrix with fading coefficients \mathbf{A} , whose diagonal elements are represented in vector \mathbf{a} .

4 Experiments

This section shows the results of the Monte Carlo simulations carried out to evaluate performance of the proposed FPA-based multiuser detector. Unless otherwise specified, a synchronous scenario with additive white Gaussian noise will be considered. In addition, Binary Phase Shift Keying (BPSK) modulation, rectangular chip pulse $\gamma(t)$ -see Eq (2)-, and Gold sequences (with $N =$

31 chips) as signatures $s_i(t)$, are used. Besides, time variation is defined by a Doppler frequency $f_d = 1000$ rad/sec. By default, $U=10$ simultaneously transmitting users are considered. Proposed FPA uses $MaxIter= 2,000$ iterations, $NumFl= 25$ flowers, and is implemented with a probability of change that depends on the Shannon entropy of the fitness values of the population as explained in step 6 of section 3.2.

4.1 Bit Error Rate (BER) vs Signal-to-Noise Ratio (SNR)

Considering user No. 1 as the user of interest (UOI), Fig. 3 shows its bit error probability as a function of the signal quality, which is evaluated in terms of $SNR_{user.1} = E_1/N_0 \in \{7, \dots, 14\}$ dB, for different conventional (decorrelator RLS (Lim et al., 1998), matched filter (Lupas & Verdu, 1989), MMSE RLS) and nature-inspired (Standard GA, Mahalanobis RBF (San-Jose-Revuelta & Cid-Sueiro, 2003), Tabu Search, Simulated Quenching and proposed FPA) algorithms. The RBF detector is implemented with the Mahalanobis distance as kernel function. Powers of the remaining users are set with a near-far effect of 4 dB (i.e. $E_j/E_1 = 4$ dB, $2 \leq j \leq U$).

Table 1 shows the mean values and the corresponding standard deviations (SD), associated to Fig. 3 plots. It is observed that SD values decrease as SNR becomes higher, and algorithms showing better convergence have a lower SD. Table 2 shows the best (minimum), mean, worst (maximum) and SD values for $SNR = 11$ and 14 dB cases.

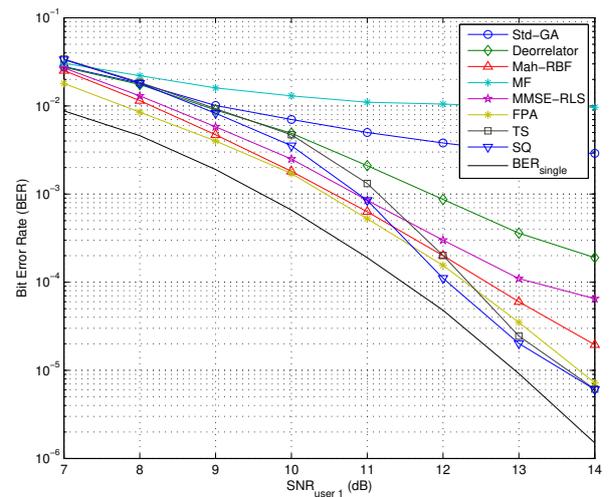


Fig. 3 Performance in terms of the probability of bit error (BER) depending on the SNR (dB) for several conventional and nature-inspired multiuser detectors. Near-far distortion: $E_j/E_1=4$ dB for $j \geq 1$. n_p : population size of the standard GA.

	$SNR_{user\ 1}$ (dB)							
	7	8	9	10	11	12	13	14
Standard GA	0.028 ± 0.012	0.018 ± 0.004	0.0101 ± 0.002	0.007 ± 0.019	0.005 ± 0.001	0.0038 ± 0.002	0.003 ± 0.0014	0.0029 ± 0.0009
Matched Filter	0.031 ± 0.009	0.022 ± 0.009	0.016 ± 0.009	0.013 ± 0.004	0.011 ± 0.002	0.011 ± 0.001	0.009 ± 0.0012	0.009 ± 1.1 × 10 ⁻³
Decorrelator RLS	0.027 ± 0.005	0.017 ± 0.002	0.009 ± 0.001	0.005 ± 0.001	0.002 ± 0.0010	0.0009 ± 1.4 × 10 ⁻⁴	3.6 × 10 ⁻⁴ ± 0.9 × 10 ⁻⁴	0.0002 ± 7 × 10 ⁻⁵
Mahalanobis RBF	0.025 ± 0.005	0.011 ± 0.0019	0.0047 ± 0.0020	0.0018 ± 0.0011	0.0006 ± 0.0002	2.0 × 10 ⁻⁴ ± 1.1 × 10 ⁻⁵	6.0 × 10 ⁻⁵ ± 1.2 × 10 ⁻⁵	1.9 × 10 ⁻⁵ ± 0.18 × 10 ⁻⁵
MMSE-RLS	0.027 ± 0.010	0.013 ± 0.005	0.006 ± 0.001	0.0025 ± 0.0011	0.0008 ± 0.0001	3.0 × 10 ⁻⁴ ± 7.9 × 10 ⁻⁵	1.1 × 10 ⁻⁴ ± 6.9 × 10 ⁻⁵	6.5 × 10 ⁻⁵ ± 2 × 10 ⁻⁵
Proposed FPA	0.018 ± 0.012	0.0085 ± 0.0011	0.004 ± 0.0007	0.0017 ± 0.0006	5.2 × 10 ⁻⁴ ± 1.3 × 10 ⁻⁴	1.5 × 10 ⁻⁴ ± 1.6 × 10 ⁻⁵	3.5 × 10 ⁻⁵ ± 0.8 × 10 ⁻⁶	7.2 × 10 ⁻⁶ ± 0.9 × 10 ⁻⁶
Tabu Search	0.034 ± 0.005	0.018 ± 0.003	0.0093 ± 0.002	0.005 ± 0.0008	0.0013 ± 0.0006	0.0002 ± 2 × 10 ⁻⁵	4.4 × 10 ⁻⁵ ± 2.2 × 10 ⁻⁶	6.1 × 10 ⁻⁶ ± 1.4 × 10 ⁻⁶
Simulated Quenching	0.033 ± 0.015	0.018 ± 0.011	0.008 ± 0.0015	0.0035 ± 0.0007	8.5 × 10 ⁻⁴ ± 1.0 × 10 ⁻⁴	1.1 × 10 ⁻⁴ ± 1.4 × 10 ⁻⁴	2.0 × 10 ⁻⁵ ± 8 × 10 ⁻⁶	6.1 × 10 ⁻⁶ ± 1.1 × 10 ⁻⁶

Table 1 Probability of bit error (Mean ± stdev) as a function of the SNR (dB) for various conventional and nature-inspired multiuser detectors. 50 independent runs. Near-far effect: $E_k/E_1=4$ dB for $k \geq 1$.

	$SNR_{user\ 1}$ (dB)							
	11				14			
	Best	Mean	Worst	SD	Best	Mean	Worst	SD
Standard GA	2.54 × 10 ⁻³	5.01 × 10 ⁻³	7.88 × 10 ⁻³	0.1 × 10 ⁻²	2.98 × 10 ⁻⁴	2.91 × 10 ⁻³	4.98 × 10 ⁻³	0.9 × 10 ⁻³
Matched Filter	7.18 × 10 ⁻³	1.10 × 10 ⁻²	1.59 × 10 ⁻²	0.2 × 10 ⁻²	5.92 × 10 ⁻³	9.65 × 10 ⁻³	1.23 × 10 ⁻²	1.1 × 10 ⁻³
Decorrelator RLS	1.01 × 10 ⁻⁴	2.12 × 10 ⁻³	3.92 × 10 ⁻³	0.1 × 10 ⁻²	7.73 × 10 ⁻⁵	1.92 × 10 ⁻⁴	3.25 × 10 ⁻⁴	7.0 × 10 ⁻⁵
Mahalanobis RBF	1.11 × 10 ⁻⁴	6.31 × 10 ⁻⁴	1.13 × 10 ⁻³	2.0 × 10 ⁻⁴	1.46 × 10 ⁻⁵	1.95 × 10 ⁻⁵	2.32 × 10 ⁻⁵	0.18 × 10 ⁻⁵
MMSE-RLS	5.63 × 10 ⁻⁴	8.51 × 10 ⁻⁴	1.06 × 10 ⁻³	1.0 × 10 ⁻⁴	1.52 × 10 ⁻⁵	6.52 × 10 ⁻⁵	1.12 × 10 ⁻⁴	2.0 × 10 ⁻⁵
Proposed FPA	2.15 × 10 ⁻⁴	5.25 × 10 ⁻⁴	7.98 × 10 ⁻⁴	1.3 × 10 ⁻⁴	5.42 × 10 ⁻⁶	7.22 × 10 ⁻⁶	9.33 × 10 ⁻⁶	0.9 × 10 ⁻⁶
Tabu Search	1.11 × 10 ⁻⁴	1.32 × 10 ⁻³	2.63 × 10 ⁻³	6.0 × 10 ⁻⁴	2.86 × 10 ⁻⁶	6.12 × 10 ⁻⁶	9.69 × 10 ⁻⁶	1.4 × 10 ⁻⁶
Simulated Quenching	6.72 × 10 ⁻⁴	8.50 × 10 ⁻⁴	1.08 × 10 ⁻³	1.0 × 10 ⁻⁴	3.81 × 10 ⁻⁶	6.05 × 10 ⁻⁶	8.74 × 10 ⁻⁶	1.1 × 10 ⁻⁶

Table 2 Best (minimum), mean, worst (maximum) and SD values of the Probability of Bit Error (BER) for different conventional and nature-inspired multiuser detectors. $SNR = 11$ and 14 dB cases. Results after 50 independent runs.

Comparing the proposed FPA to a standard GA-based MUD, it can be seen how FPA performance is close to the theoretical single user limit –notice that a lower limit for a multiuser detector algorithm is obtained simulating the probability of error when $U = 1$ (absence of interfering users) (Proakis, 1998)– while the GA-based MUD has difficulties to converge as SNR increases, even with a higher number of iterations and generations. This situation was already observed by Shayesteh et al. (2003).

Comparison with other nature-inspired algorithms such as simulated quenching (SQ) and tabu search (TS) show similar results whenever user 1 (UOI) transmits with high enough power ($SNR_{user\ 1} \gtrsim 12$ dB) –see Fig. 3. In order to get a fair comparison, SQ and TS were implemented with an equivalent computational load measured in terms of fitness function evaluations. This is achieved by properly setting the number of iterations. Results are mean values after 50 independent runs. When $SNR_{user\ 1} < 12$ dB, both SQ and TS fail to achieve acceptable estimates, while FPA still works correctly. The reason for this is that FPA combines more efficiently the explorative/exploitative sense of search, in part thanks to the adaptation of the probability of change, which controls local/global pollination, by monitoring the entropy of the population fitness, as explained in section 3.2.

Performances of some traditional multiuser detectors, such as Matched Filter, Minimum Mean Square Error Estimator and decorrelator detectors, are also shown in Fig. 3. Results indicate that simple algorithms (e.g. standard GA and MF) cannot properly converge to good data estimates even when user 1 transmits with high power. On the other hand, nonlinear algorithms (RBF,

MMSE and decorrelator) lead to an intermediate BER between SQ, TS and FPA, and that achieved with MF or standard GA.

Finally we have compared computation time between of proposed FPA, Std-GA, TS and SQ. Every algorithm is iterated until the same bit error probability (BER) is achieved. Results are shown in Table 3. Table cells show percentage values normalized with respect to the time required by proposed FPA, whose time is shown as 100%.

Processing time			
Proposed FPA	Std-GA	TS	SQ
100 %	445 %	150 %	155 %

Table 3 Comparison of processing time required to achieve the same BER. Percentage values normalized with respect to the time required by proposed FPA.

We see that Std-GA requires about $4 \times$ time to achieve the same BER, while TS and SQ are in the order of $1.5 \times$.

4.2 Statistical tests

In order to evaluate the statistical significance of results involving nature-inspired algorithms, we performed a Kruskal-Wallis test with a Bonferroni correction. This test establishes if performance differences between Std-GA, SQ or TS, compared to proposed FPA, are statistically significant. Table 4 shows the results when a significance level $\alpha = 0.05$ is used.

In accordance to (Črepinšek, Liu & Mernik, 2014), algorithms are configured so as to have equal compu-

	SNR_{user1} (dB)							
	7	8	9	10	11	12	13	14
Standard GA	†	†	□	□	□	□	□	□
Tabu Search	□	□	□	□	□	†	□	†
Simulated Quenching	□	□	□	□	□	□	□	†

Table 4 Statistical significance of results using a Kruskal-Wallis test with Bonferroni correction ($\alpha = 0.05$). Each row compares proposed FPA against Std-GA, TS or SQ, respectively. □: FPA wins, †: difference is not statistically significant, ∘: Std-GA/TS/SQ wins.

tational load in terms of fitness function evaluations. In particular, the number of generations is adjusted so as to have the same number of fitness function evaluations than FPA for each cell of Table 4.

Next, we compare performances of four metaheuristics using the Friedman test (Friedman, 1940). Proposed algorithm is compared to Std-GA, TS and SQ. The problem test suite consists of 8 optimization problems, each one defined with a specific value of $SNR_{user1}(dB) = \gamma$ dB, $\gamma \in [7, 14]$. The Friedman test is conducted to detect significant behaviour differences between two or more detectors. Overage rankings are shown in Table 5. The best average ranking (lowest value) is in italic and corresponds to the FPA approach, which outperforms the other three schemes. A p -value of 0.0091 is obtained showing that there exist significant differences between the behaviour of the four algorithms.

Algorithm	Friedman Test Score
FPA	<i>1.625</i>
SQ	1.875
TS	3.00
Std-GA	3.50

Table 5 Average ranking achieved with the Friedman test for four nature-inspired multiuser detectors: proposed FPA, standard GA, Tabu Search and Simulated Quenching. Each test problem comes given by a specific $SNR_{user1}(dB)$ value, from set {7, 8, 9, 10, 11, 12, 13, 14}.

4.3 Bit Error Rate vs number of active users (capacity)

An important parameter used to assess the quality of a multiuser detector is its *capacity*. Capacity plots show the probability of erroneous symbol detection depending on the number of active system users. Numerical results are plotted in Fig. 4, where the error probabilities of several detectors (FPA, RBF, GA, SQ, TS, MMSEE and MF) are compared.

In this simulation no near-far effect is present ($E_i = E_j$, $1 \leq i, j \leq U, i \neq j$). It can be seen how, on the basis of the BER metric, MF and MMSE methods are clearly

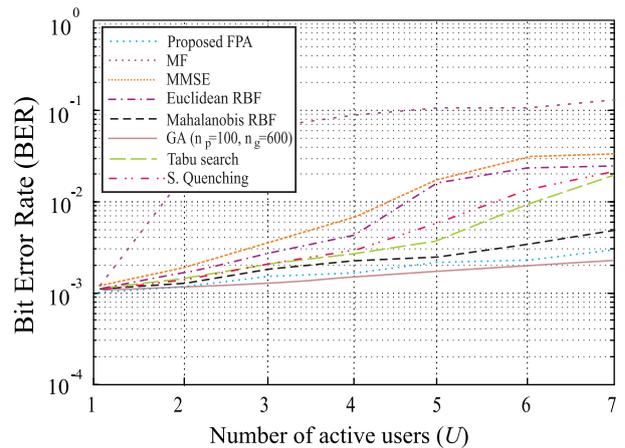


Fig. 4 Bit Error Rate vs number of active users U . $E_i = E_j$, $1 \leq i, j \leq U, i \neq j$. GA is implemented with $n_p = 100$ individuals and $n_g = 600$ iterations. Average values after 30 independent runs.

outperformed by Mahalanobis-RBF and nature-inspired schemes. Under these conditions, multi-access interference greatly affects performance of MF, while the MMSE detector is not able to establish a good enough decision boundary. Notice that RBF implemented with the Mahalanobis distance yields better results than with the Euclidean one.

Performances of nonlinear schemes (the proposed FPA, Mahalanobis RBF and GA) are very close. However, it must be pointed out that both RBF algorithms involve a much higher computational load (proportional to 2^U) and require a supervised initial training period (San-Jose-Revuelta & Cid-Sueiro, 2003).

On the other hand, we see that when the number of active users is moderate, performances of GA, SQ, TS, Mahalanobis-RBF and FPA are, all of them, very similar. However, as the number of users increases, performance drops close to those of the MMSE and Euclidean-RBF schemes, since multi-access interference is more relevant.

It is obvious that performance degrades as U increases, since population, in every evolutionary algorithm, is finite, and these methods are not able to properly explore the solutions space. A notable advantage of FPA and evolutionary approaches, in contrast to conventional search methods, is that they do not need any memory elements, since information related to previous iterations does not need to be stored.

4.4 Channel estimation performance

Accuracy of fading coefficients' estimates is next analyzed. Performance is evaluated using the mean squared error (MSE) of the solution represented by the fittest

flower when FPA finishes (\mathbf{b}^*). Fig. 5 shows comparative results with three different detectors: a Bayesian MUD proposed by San-José-Revuelta (2007) and the so-called “MAP-GCGS” and “MAP-GS” algorithms developed by Huang & Djuric (2002), for several E_k/N_0 ratios measured along a 100-long symbols frame. The number of users is set to $U = 8$, all of them transmitting with the same energy.

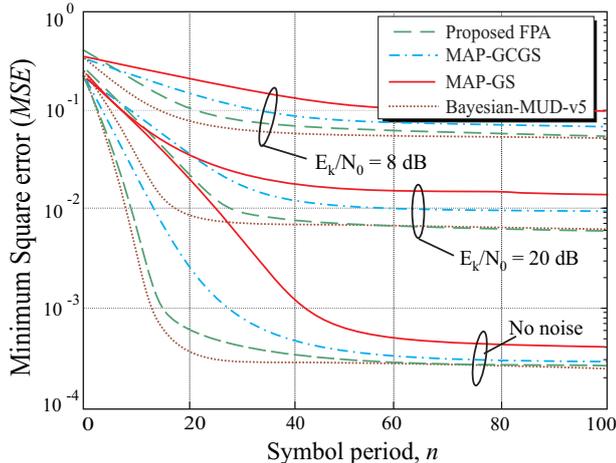


Fig. 5 Average mean squared channel estimation error over a period of 100 symbols. The energy received for every active user is the same. Averaged values in 75 runs.

Numerical results show that, on average, the proposed FPA requires about 30-40 symbol periods to converge close to the final value, while MAP-GS and MAP-GCGS require about 40-60 samples, and the Bayesian-MUD-v5 detector takes a shorter period of about 25-30 samples.

It must be noticed that both Bayesian-MUD-v5 and FPA achieve a lower mean square error. However, the Bayesian scheme does not explicitly estimate the channel response coefficients, which the FPA method does. Besides, another advantage of nature-inspired methods like FPA is that they do not require any supervised initial period at all. Conversely, both MAP-GS and MAP-GCGS must be implemented with a 18 samples length burn-in period.

Next, Fig.6 compares the Mean Square Error (MSE) of the proposed method to five different detectors: the conventional decorrelator estimator (Lim et al., 1998), the Bayesian MUD v.5 (San-José-Revuelta, 2007), the MAP-GS algorithm (Huang & Djuric, 2002), the MCRB-U algorithm (Farhang-Boroujeny et al., 2006) and the GA-based detector (Yen & Hanzo, 2001). Error is represented as a function of the normalized symbol energy of the users ($E_i = E_j, i \neq j, 1 \leq i, j \leq U$). The

single-user limit is plotted in black as a reference, as well.

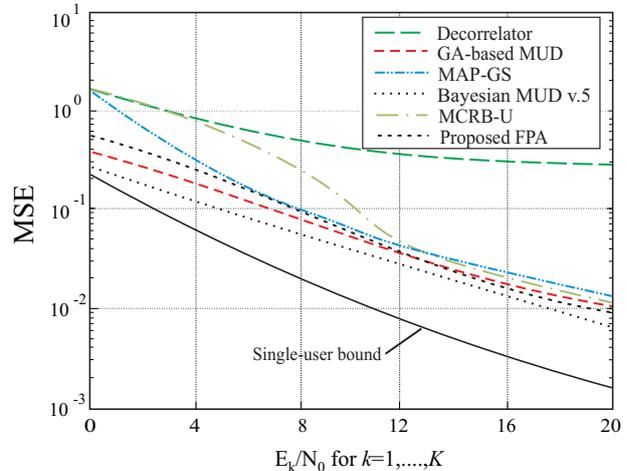


Fig. 6 Average mean squared channel estimation error as a function of the users' bit energy. $U = 8$. $E_i = E_j, i \neq j, 1 \leq i, j \leq 8$. Every active user transmits with the same power.

Results show how nonlinear estimators, such as the GA-based MUD, the Bayesian MUD v.5 and the proposed FPA, present an importantly lower MSE than those of the decorrelator detector and the Bayesian approaches used for comparison (MAP-GS and MCRB-U), with a performance near the single-user limit, particularly for low and moderate SNRs (< 11 dB). Performances of FPA and Bayesian schemes are very similar when $E_k/N_0 \geq 12$ dB. However, for low SNR values, on the basis of the MSE metric, Bayesian MUD v.5, GA and FPA detectors outperform the other algorithms.

4.5 Near-far performance

The near-far effect considers the situation in which part of the interfering users are closer to the base-station than the user of interest (UOI) is. This involves that their signals are received with stronger power than the one coming from the user of interest, making, this way, more difficult the problem of satisfactorily extracting useful data (Proakis, 1998; Verdu, 1998). Bit Error Rate of the UOI as a function of the difference of received power between the UOI and the other users is evaluated. Three interfering users, i.e. $U = 4$, processing gain $N = 32$ and noise variance $\sigma_n^2 = 0.5$, are considered. Besides, powers from the three interfering users are the same at the receiver antenna. In Fig. 7 the BER of the UOI versus E_1/E_k ($2 \leq k \leq U$) is shown for several detectors. Error probabilities of two conventional detectors –the decorrelator (Lim et al., 1998) and

the MF scheme (Lupas & Verdu, 1989)– are shown for comparison, as well as the performance of a multistage detector (Buehrer & Woerner, 1996) and the MAP-GC Bayesian scheme (Huang & Djuric, 2002).

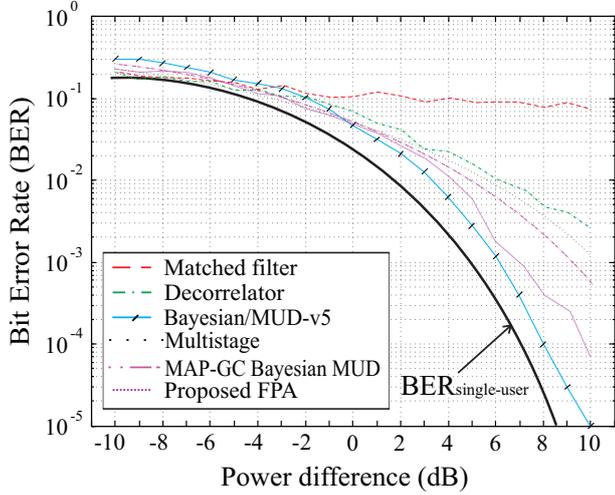


Fig. 7 Bit Error Rate of the user of interest (UOI) vs SNR for different multiuser detectors. Average results after 30 independent runs.

For this simulation, intersymbol interference is assumed by considering that the transfer function of the UOIs channel response is $H_{\text{UOI}}(z) = a_0 + 0.471z^{-1} + 0.063z^{-2}$. The first coefficient (a_0) varies in such a way that signal-to-interference ratio is $-10 \leq \text{SIR} \leq 10$ dB. We can see how BER decreases as power difference increases, whatever detector scheme is used, which makes sense, since the received signal from the UOI becomes stronger at the receiver as both a_0 and the power difference with respect to interferers, increase.

Results show that both Bayesian-MUD-v5 (San-José-Revuelta, 2007) and proposed FPA are near-far resistant, achieving a bit error rate close to the single user limit, whilst MF shows a notable degradation –it is well-known that interferences limit its performance. On the other hand, on the basis of the BER metric, the decorrelator detector has a performance somewhat outperformed by MAP-GC algorithm and multistage detectors.

If computational load is analyzed by fixing $U = 4$, $\sigma_n^2 = 0.5$ and a power difference of 2 dB, it can be seen that the decorrelator detector and proposed FPA have a nice trade-off between complexity and performance (decorrelator takes about $1.3 \times$ the time of FPA). FPA shows good efficiency, close to the Bayesian scheme, with only a small BER degradation, while demanding less computational load ($1.8 \times$ of FPA). On contrast, the MAP-GC algorithm is the most complex and time-consuming algorithm ($2.5 \times$ of FPA).

Finally, Fig. 8 shows the near-far performance of the proposed FPA detector depending on the BER of the UOI. The averaged received symbol energies of the remaining users are adjusted 0, 5, 10 and 15 dB higher than the UOI's energy. Numerical results show that when $5 \leq E_k/E_1 \leq 10$ dB, the proposed FPA has good near-far capabilities up to $E_1/N_0 \approx 26$ dB. Higher values of E_1/E_0 lead to a slight BER performance degradation.

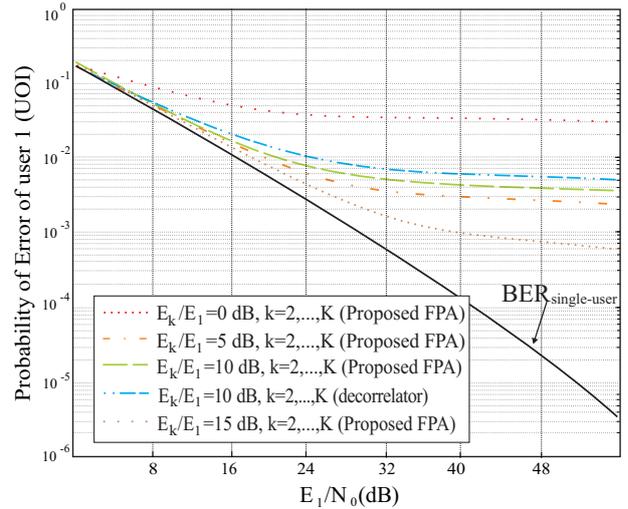


Fig. 8 Bit Error Rate performance for $K = 10$ active users with $E_k/E_1 = 0, 5, 10$ and 15 dB for $k = 2, \dots, U$. User 1: user of interest (UOI).

The BER plot corresponding to a conventional decorrelator detector is shown for $E_k/E_1 = 10$ dB to allow comparison, as well. It can be seen how, on the basis of the probability of bit error of the UOI metric, performance drops a little with this method with respect to the FPA scheme.

Consequently, we can say that the proposed FPA multiuser detector is near-far resistant and robust when both inter-symbol and multi-access interferences exist, constituting an interesting alternative to already existing methods.

5 Conclusions

In this work, a multiuser detector based on FPA has been developed to jointly solve the problems of symbol detection and channel estimation in a DS/CDMA multiuser communication environment, which is one of the most important and implemented channel sharing techniques in advanced communication systems, such as some parts of LTE and 5G standards.

Among the improvements introduced over the standard FPA are: scale factors associated with Levy flight and change probability are adjusted according to the entropy of the aptitude of the flowers so that an optimal compromise between exploitation and exploration is kept. Besides, two different parts are considered within the encoding of each flower, and the pollination processes are applied differently to each part.

The algorithm obtained in this way is implementable in real CDMA systems (in LTE, GPS, IS95, 5G, etc.), allowing to work in more critical conditions (lower received power, greater interferences, large fading, higher number of users) and at higher transmission rates, while keeping conceptual and computational simplicity.

Numerical simulations have been carried out, as far as we know, in fair terms, including comparisons with several powerful and well-known conventional schemes as well as with other metaheuristics. The proposed method is proved to be an efficient tool to solve the complex problem of equalization in digital communications. FPA offers competitive performance, especially when conditions become more difficult (a large number of interfering users, a low SNR of the user of interest or the existence of near-far effects) and less complexity than that of conventional methods achieving similar performance. When inter-symbol and multiple-access interferences are not high, other nature-inspired schemes, such as TS, GA and SQ, show similar performance to those of the proposed detector and the optimal receiver. But when interferences become stronger, or the power of the user of interest is lower, proposed FPA performs better, in part due to its powerful search capabilities that adjust the diversity of the population by monitoring the entropy of the population fitness. This way, the probability of change, which selects between local and global pollination, is in-service adjusted using this information. Finally, several tests have been run to analyze the statistical significance of results.

Among the limitations of our work, we highlight the difficulty and dependence of the optimal initial adjustment of parameters. Besides, the paper is focused on a binary source alphabet and considers a perfectly synchronous system.

As future lines of work we consider the extension to non-binary modulations, asynchronous systems and other channel sharing schemes. Likewise, our next works will be aimed at the detailed study of the influence of each parameter of the algorithm, a more detailed study of the computational load and a deeper statistical analysis of the results.

A Particularization to a chip-rate algorithm

In this case, taking into account that the energy of the chips is $\mathcal{E}_\gamma = \int_0^{T_c} |\gamma(t)|^2 dt$, a sequence of N samples is obtained for each symbol, whose components are calculated as

$$\begin{aligned} r_{n,j} &= \int_{nT+jT_c}^{nT+(j+1)T_c} r(t)\gamma(t-nT-jT_c)dt \\ &= \sum_{u=1}^U a_u(n)d_u(n)s_u(j)\mathcal{E}_\gamma + \int_0^{T_c} g(t+nT-jT_c)\gamma(t)dt, \\ & \quad j = 0, 1, \dots, N-1 \end{aligned} \quad (19)$$

This set of N samples is normalized by the chip energy \mathcal{E}_γ and grouped into a vector \mathbf{r}_n^{chip} . Thus, we can write

$$\mathbf{r}_n^{chip} = \sum_{i=1}^U \mathbf{s}_i a_i(n) d_i(n) + \mathbf{g}(n) = \mathbf{S} \mathbf{A} d(n) + \mathbf{g}(n) \quad (20)$$

where $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_U]$ is the $N \times U$ matrix whose columns contain the users' signatures \mathbf{s}_i , and $\mathbf{g}(n) = [g_{n,0}, g_{n,1}, \dots, g_{n,N-1}]^T$ stands for the normalized noise vector with components

$$g_{n,j} = \frac{1}{\mathcal{E}_\gamma} \int_0^{T_c} g(t+nT+jT_c)\gamma(t)dt, \quad j = 0, 1, \dots, N-1 \quad (21)$$

Since \mathbf{g} represents a zero-mean, white and Gaussian noise process, its covariance matrix is $E\{\mathbf{g}(n)\mathbf{g}(n)^H\} = \sigma^2 \mathbf{I}_N$, with \mathbf{I}_N being the $N \times N$ identity matrix.

B List of acronyms

The main acronyms used in the paper are:

- ACO: Ant colony algorithm
- BPSK: Binary Phase Shift Keying
- CSO: Cat swarm optimization
- DS/CDMA: Direct Sequence / Code-Division Multiple-Access
- FPA: Flower pollination algorithm
- GA: Genetic algorithm
- GPS: Global positioning system
- ISI: Intersymbol interference
- MAI: Multiaccess interference
- MF: Matched filter
- ML: Maximum Likelihood
- MMSEE: Minimum Mean Square Error Estimator
- MUD: Multiuser detector
- PSO: Particle swarm optimization
- RBF: Radial basis function
- SA: Simulated annealing
- SNR: Signal-to-noise ratio
- SQ: Simulated quenching
- TS: Tabu search UOI: User of interest

Compliance with ethical standards

Conflict of Interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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