Armington Elasticity and Development

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ABSTRACT

We model trade competitiveness over the course of economic development, by focusing on real world links between technology, market shares, and consumers’ demand for technology products. Our starting point is the Armington Elasticity (AE) which is the standard way to represent region-specific preferences in international trade, however a great drawback is that it is assumed fixed. Our paper contributes by modelling the role of technological and economic progress in evolution of AE. This framework has theoretical, and policy applications. We provide illustrations of how the framework can be used to analyze the conditions for successful government policy interventions in trade. Specifically we model supply and demand sides: 1) technological progress influencing AE, as substitution becomes easier when varieties embody similar technology and 2) steadier demand for technology intensive products as the economy develops and incomes rise.

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KEYWORDS: Armington elasticity; trade policy; economic development; technological progress.

Armington (1969) presents a theory to extend the concept of price elasticity of demand for products to the demand for varieties of each product. The varieties of a product are differentiated based on the producing country. Each country produces a different variety. The Armington theory yields tractable solutions for the own and cross price elasticity of demand for each variety, j, of every product i. This turns out to be a function of the price elasticity of product i, the elasticity of substitution among varieties of i-th product (the Armington Elasticity), and the market share of variety j. Armington argues that distinguishing varieties of the same product based on the country of production is realistic.
So, an interesting question is what causes the differences among varieties of the same product produced by different countries? It is an important policy question, as well as an important question from a trade theory perspective. The Armington approach is used widely in CGE models, where it is the standard methodology for representing region-specific preferences for goods in international trade. With deadlocks in multilateral trade negotiations under the WTO (Baldwin, 2016), and looming trade wars (Regan & Barrett, 2019), a useful role for trade literature is to update trade policy with a focus on the potential competitive impact of liberalization of trade. The degree to which imported and domestically produced goods substitute for one another is central to such an analysis. A study of the evolving values of the parameters representing the rate at which this substitution occurs, for countries at different levels of development, is thus highly relevant.

Our paper is novel because so far all the causes of Armington-type imperfect substitution, between varieties of a product, explored in the literature (Section I, discusses this literature in detail) are immediate or lagged outcomes of trade/investment/labor policy, physical distance, and have little or nothing to do with the intrinsic characteristics of the product itself. By contrast, we focus on intrinsic characteristics of the product stemming directly from the technological capability of the producing country. Trade competitiveness will also depend on consumer preferences. Therefore to complete the competitive landscape, on the demand side, we account for changing price elasticities of demand as per capita incomes rise with technological progress.

This approach seems to be supported by the availability of extremely detailed trade data that has made it possible to investigate if there is measurable difference among products based solely on their country of origin. For example, Schott (2004) finds that countries export the same product (at the 10 digit level of the Harmonized System of product classification, therefore they are listed as “identical” products) at vastly different unit-values depending on their per capita incomes. This raises the possibility that countries specialize in different quality levels (varieties of the same product) based on their technological capabilities.
The theoretical contribution of this paper is to consider the possibility of genuinely different quality of production based on available technology (Romer (1994), Fagerberg (2000), Wolff (2000), Mukerji (2009 and 2013)). In this paper we consider long term technological development causing, and changing Armington-type imperfect substitution among varieties of a product. The only other work that addresses production processes which lead to intrinsically different product characteristics is Welsch (2006) which attributes the falling AE in France in the 1990’s to the evolution of a more specialized production structure that ceased production of close substitutes for many imports. Even there the outcome is studied as a lagged response to trade liberalization policy.

Armington (1969) found that price elasticity of a variety of a product is a function of the AE and the overall product price elasticity, both of which are assumed fixed. In this paper we extend the Armington (1969) model so that both these parameters are allowed to change with development and technological progress. Specifically, technological progress influences AE on the supply side by improving the quality of skill/capital/technology intensive production in an economy, making the domestic variety a closer substitute for the imported variety. And on the demand side, improvement in technology is associated with rising per capita income (Schott 2004, Balassa 1979, Romalis 2004, Hallak 2006, Barro 1991, Linder 1961), which leads to steadier demand for skill/capital/technology products. We use the term steady demand to signify decreasing price elasticity of demand with respect to per capita income. We utilize a modified version of the Markusen (2013) model to reflect this in the second parameter, the overall product price elasticity. Bringing together these developments on the supply and demand side, along with market shares of each variety, our paper yields a relationship between price elasticity of demand for domestic and imported varieties of each product and the level of economic development of a given country. We use numerical simulation to derive the form of this relationship.

Our model has widespread policy application. We focus on the intersection between market shares, technological innovation and consumers’ evolving preference for technology intensive products. A commonplace example is of governments, over time and in economies at different
stages of development, attempting to support domestic producers by influencing their market shares, as well as by supporting technological innovation. An interesting contrast can be drawn of the impact of this policy in a country where technology lags the rest of the world versus where the country is at or above the rest of the world’s technology level. We discuss two such examples now.

Our example of a country below the existing world technology frontier at that time, is the Indian consumer automobile industry in the second half of the twentieth century. A contrasting example we provide is the production of electric car batteries in Asia where producers are already at the world technology frontier.

In our first example, before the trade liberalization of 1991 the Indian government encouraged import substitution through prohibitive tariffs and import licenses. The government’s policy ensured large market shares for domestic producers and was supposed to encourage home grown technological innovation (Bhagwati and Desai, 1970). The unintended side effect was often technological stagnation, perhaps driven by lack of competition and limited exposure to emerging technologies through imports (Bhagwati and Srinivasan, 1975). For instance in the consumer automobile market, cars were limited largely to just two vehicles – Hindustan Ambassador and Premier Padmini - both manufactured domestically with the original design licensed/bought from British Motor Corp. and Fiat respectively (Sumantran, 1993). The cars and the technology behind them remained practically unchanged throughout their reign of the Indian market from the 1960’s and 1970’s to when they finally lost practically all their market shares once Indian trade was liberalized. In our model we present one scenario (Section II.4) where only demand side forces act and the supply side is held constant, this could be a framework to analyse situations like the Indian economy’s example. We present this analysis in Section III.1 and Section III.2.

Our second example is a contrast to the first and presents another policy application of this model in the market for electric cars, where an area of intense competition is the European market for lithium-ion car batteries for these cars. The European Commission is seeking to advance local companies in battery production in Europe (Onstad, et.al., 2018). While Asian manufacturers, who
currently lead in market share, are seeking to locate production facilities in Europe where electric
car manufacturing has accelerated and buying batteries from abroad is now less feasible. For now it
appears to be a losing battle for the European companies, as European governments are unlikely to
match the huge production subsidies that Asian governments provide (Stanway, 2018). The best
hope for European companies is to invest in the next generation “solid state” batteries where they
will not be encumbered by the pre-existing market leadership of Asian rivals. Our model, in the
case where both supply and demand side forces act (Section II.5) but one variety has a very large
market share, presents a framework to analyse such cases as this example. We present this analysis
in Section III.1.

We can see from the above examples that technology, market shares and consumers’
demand for advanced products are closely linked in practice, with market shares sometimes
trumping even technological prowess, e.g., in the lithium-ion batteries market. In other instances,
like the case of India outlined above, market share is not enough to ensure dominance and the entry
of superior technology changes the competitive landscape quickly.

The rest of the paper is laid out as follows. Section I contains a literature review, Section II
presents the theoretical model and Section III, the simulation results. Section IV concludes.

I. LITERATURE REVIEW

Table 1 presents the source of AE estimates along with the sample of countries and years on
which each estimate is based. In addition, there is a column with the share of patent applications
filed by the countries in the sample as a percentage of world totals to measure technological
development. The aim of Table 1 is to show a comprehensive list of AE estimates in the literature
and it demonstrates standard patterns: AE estimates are higher for disaggregated data, for longer
term data and for cross section data (e.g. McDaniel and Balistreri (2002), Ruhl (2003)). In this
work, however we would like to draw attention to two new characteristics of the estimates that will
clarify the contribution we seek to make to the literature. First, we separate AE estimates based on
whether they measured substitutability between domestic versus imported products or only among
imported products. Viewed in this light, the table reveals a new pattern. We see that this separation makes it clear that imported products are easier to substitute for one another, compared to their substitutability with the domestic variety. This is evidenced by the AE estimates being on average larger when measuring substitutability among only imported products. This has been noted by Feenstra et al. (2012) whose estimates are also part of Table 1. However the reasons for this explored by Feenstra et al. (2012) are not intrinsic product characteristics, instead they consider imperfect substitutability based on factors such as, political processes/foreign ownership which make consumer perception of products become more or less dependent on the product’s country of origin and. For example, foreign ownership might lead to more openness, and rising AE. Conversely, unionization might lead to falling AE due to political movements that encourage people to buy domestic, for example, in an attempt to protect domestic jobs (Blonigen & Wilson (1999)).

The contribution of our paper is to explore technological development creating real differences between domestic and imported product characteristics and thus affecting substitutability. To measure technology on Table 1, we add a column of patent applications as a percentage of total world patent applications to measure countries’ technological development. However, it is difficult to discern any broad patterns from the addition of the patent data. This is not surprising since the estimates of AE available in the literature are influenced greatly by the trade policy. For example a small change in prices can bring about a large trade response due to bilateral, regional or multilateral trade liberalization. Thus the countries included in the estimation will influences the AE estimate due to trade policies that were directed towards these countries. In addition, particular industries analysed may have characteristics e.g., trade restrictions, consumer perceptions based on unionization, political processes, etc. that influence the AE estimated. Furthermore, as noted above, the characteristics of the data such as aggregation level, time period covered, cross section/time series nature also impact the AE estimate.

Our contribution in this paper is to model how the process of development, particularly the availability of technology will shape the AE between the domestic and imported goods. One of the
important questions this will help answer is how economic fundamentals of each economy might explain why AE estimates of domestic versus imported goods is so much lower than that among imported goods alone. This important aspect has so far been neglected by the literature.

II. THEORETICAL MODEL

II.1 Supply Side: Technology impacts skill/capital/technology intensive production

We use a modified version of the Armington (1969) model. Our model’s novel contribution on the supply side is that unlike extant literature, which assumes fixed AE, we allow AE to be a function of technology. Technology is an important determinant of product quality, especially in developing countries. For instance Romer (1994) and Mukerji (2009 and 2013) argue quality improvement eases substitution between imports and domestic produced varieties in skill/capital/technology intensive products. Thus in our model, AE is directly proportional to the level of existing production technology. This represents the increasing ease of substitution between home and imported varieties as the home technology progresses towards the world technology frontier.

This contribution is also very timely in today’s technology driven competitive landscape, where ignoring the differences in technology content of the varieties of a given product misses a fundamental basis of quality and competitiveness. Consider commonplace products of everyday use like cellular phones and electric cars – each have varieties that compete based on a constant race for technological superiority. This is also in keeping with the large theoretical and empirical literature on the international competitiveness of product varieties originating from countries at different levels of technological progress (Schott (2002, 2004), Mukerji (2009, 2013)).

We model AE as a function of the measure of technological development denoted by the index, $T$, that varies from 0 to 100 -

$$\sigma(T) = \sigma_0 e^{\sigma T}$$

(1)
Where AE varies from low ($\sigma_L$) to high ($\sigma_H$) levels depending on the technology available in a particular country and $r$ is a positive constant. We choose an exponential functional form since it yields an intuitively appealing characteristic of steeper response of the elasticity of substitution at higher income/technology levels. We would naturally expect higher levels of technology leading to ever faster convergence towards $\sigma_H$. Note that the form of convergence of Armington elasticity implied by (1) means that the domestic economy is a technological laggard country. In our model we have assumed $r > 0$ in (1), if instead we had chosen $r < 0$ that would imply a situation where as the level of technology increases, goods become more differentiated and hence more difficult to substitute. Our choice of the study of technology catch-up from a laggard country’s perspective is driven by the potential for timely application to the current issues in international trade competitiveness. As developing countries enter global trade in larger numbers than ever before, fears of competition are leading to rising protectionism (Baldwin, 2016). In this context, a careful study of the trajectory of technology catch-up by developing countries and implications for international competition is timely.

During our simulations presented in Section III, $T$, the technology index, is taken to be the percentile per capita income of households in the country because of the strong positive relation between per capita income and technological sophistication indicated in the literature (Schott 2004, Balassa 1979, Romalis 2004, Hallak 2006, Barro 1991, Linder 1961). We will assume in our model that all households have the same per capita income $m$ and use it to represent the percentile of per capita income to which the country belongs, indexed to range between 0 and 100 internationally. Therefore low, middle and high income countries will be denoted by the percentile of their per capita income. Later, in our simulations the same parameter is used to represent the percentile per capita income $m$, and the technology index, $T$. 

Let $T_{\text{min}}$ = the minimum technology index which when achieved by a country, makes its domestic product a robust substitute for the imported variety, thus at $T_{\text{min}}$ the AE takes on the high value of $\sigma_H$. Thus $\sigma_H = \sigma_L e^{r T_{\text{min}}}$, taking natural logs and solving for $r$, we get the relationship -

$$\sigma(T) = \sigma_L e^{\frac{r}{T_{\text{min}}} \frac{\sigma_H}{\sigma_L}}$$

(2)

From (2) we note that the speed of convergence of the Armington elasticity of a country is jointly determined by the ratio $\frac{\sigma_H}{\sigma_L}$ and the value of $T_{\text{min}}$. As per model characterization, once $T_{\text{min}}$ is achieved, Armington elasticity attains the value $\sigma_H$ and even when $T$ grows larger than $T_{\text{min}}$, $\sigma_H$ remains the Armington elasticity for the country. In other words, once at $T_{\text{min}}$, the country attains the technological progress required to produce at the world technology frontier producing close substitutes for the varieties produced abroad.

II.2 Demand Side: Per-capita income & skill/capital/technology intensive product demand

So far our theory has focused on the impact of technological change on the production side. Analysis of trade competitiveness, however, would be incomplete without the demand side. In a general equilibrium model, technological improvement is related to rising per capita income as productivity rises. In order to reflect this, we use an extended Markusen (2013) model with a Stone-Geary utility function. This set-up gives us the result that the Marshallian price elasticity of demand for skill/capital/technology intensive products is falling in per capita income.

This is an important contribution of our model as we move away from the typical trade theory assumption that consumers have identical and homothetic preferences within and across countries (Markusen, 2013) and instead incorporate the impact of evolving consumer preferences that must go hand in hand with the improvement in technology on the production side.

Thus on the demand side we use a Stone-Geary utility function. Our model has four products that are consumed in the economy: products $x_1$ and $x_2$ are relatively
skill/capital/technology intensive, thus we extend the Markusen (2013) model here, to include more
than one skill/capital/technology intensive product; product y is a relatively labor intensive product,
this product is assumed to require rudimentary technology that all countries possess, and z is an
endowment product that is non-rival and non-excludable. Each of \( x_1 \) and \( x_2 \) have \( n \) varieties, one
domestically produced and the rest are imported from \( n - 1 \) countries respectively. Household
utility is -

\[
\begin{align*}
  u(x_{11}, x_{12}, x_{13}, x_{14}, \ldots, x_{1n}, x_{21}, x_{22}, x_{23}, x_{24}, \ldots, x_{2n}, z, y) \\
  = u(x_1, x_2, z, y) = (x_1 + x_2 + z)^\beta y^{1-\beta}
\end{align*}
\]

(3)

\( \beta \) is the share of the budget spent on products \( x_1 \) and \( x_2 \) and the rest \( 1 - \beta \) is spent on
product y. The first equality where we collapse the utility function so that \( x_1 \) and \( x_2 \) each now
represent all \( n \) varieties of the product, follows from an assumption of functional separability of \( x_1 \)
and \( x_2 \). The second equality presents the Stone-Geary utility function.

Let \( m^q \) be the income of household \( q \). Let \( P_{x_1} \), \( P_{x_2} \) and \( P_y \) be the prices of \( x_1 \), \( x_2 \) and y.
The household budget constraint is -

\[
m^q = P_{x_1} x_1 + P_{x_2} x_2 + P_y y
\]

(4)

There are \( L \) households, \( M = \sum_{q=1}^{L} m^q \) and \( Z = zL \). Maximizing (2) subject to (3), yields the
following first order conditions where \( \lambda \) denotes the Lagrangian multiplier –

\[
\begin{align*}
  \beta(x_1 + x_2 + z)^{\beta-1} y^{1-\beta} - \lambda P_{x_1} &= 0 \\
  \beta(x_1 + x_2 + z)^{\beta-1} y^{1-\beta} - \lambda P_{x_2} &= 0 \\
  (x_1 + x_2 + z)^{\beta} (1 - \beta) y^{-\beta} - \lambda P_y &= 0
\end{align*}
\]

(5) (6) (7)

Solving for \( x_1 \) using equations (5) and (7) yields –

\[
x_1 = \frac{\beta(m^q - x z P_{x_2})}{P_{x_1}} + (\beta - 1)(x_2 + z)
\]

(8)
We use the aggregate demand $X_i$ to denote the sum over all households’ demand for $x_i$ and write

$$X_i = \sum_{q=1}^{L} x_q = \frac{\beta(M - X_2 p_{x_2})}{p_{x_i}} + (\beta - 1)(X_2 + Z) \quad (9)$$

Defining the price elasticity as positive and by using $\frac{dX_1}{dP_{x_i}} = -\frac{\beta(M - X_2 p_{x_2})}{P_{x_i}}$ we get -

$$\eta_{X_1} = \left[ \frac{P_{x_i}}{X_1} \frac{dX_1}{dP_{x_i}} \right] = \frac{M - X_2 p_{x_2}}{M - X_2 P_{x_2} + \frac{\beta - 1}{\beta}(Z + X_2)P_{x_i}}$$

$$= \frac{m - \bar{x}_2 p_{x_2}}{m - \bar{x}_2 P_{x_2} - \frac{\beta - 1}{\beta}(z + \bar{x}_2)P_{x_i}} \quad (10)$$

Where $m$ = average household budget, we use this to denote per capita income and $\bar{x}_2$ = average household consumption of product $x_2$. Equation (10) indicates that the price elasticity is declining in per capita income, indicating a steadier demand for $x_1$ with rising per capita income ($m$).

The minimum average household income ($m_0$), required for an average household to start consuming the skill/capital/technology intensive product $x_1$ is –

$$\frac{\beta(m - \bar{x}_2 p_{x_2})}{p_{x_i}} - (1 - \beta)(\bar{x}_2 + z) > 0$$

$$\Rightarrow m > \bar{x}_2 p_{x_2} + \frac{(1 - \beta)(\bar{x}_2 + z)P_{x_i}}{\beta} = m_0 \quad (11)$$

For each product $X_i$, $i = 1, 2$ we minimize the expenditure on the $n$ varieties bought of $X_i$ where each country supplies a unique variety. Assuming functional separability of each $X_i$ and a CES form, allows us to construct the quantity index from all the varieties of the product $X_i$,

$$X_i = \phi_i(X_{i1}, X_{i2}, ..., X_{in}) = [b_{i1} X_{i1}^{-\rho} + b_{i2} X_{i2}^{-\rho} + .... + b_{in} X_{in}^{-\rho}]^{\frac{1}{\rho}}. \text{ Here the price index } P_{x_i} \text{ is the unit} \quad \text{11}$$
expenditure function for group of products index by \( i \), i.e. all the varieties of product \( X_i \). That is

\[
P_{x_i} = e_i(P_{x_i}, \mathbf{1}) = \min_{x_i} \{ P_{x_i} x_i : \phi(X_i) = 1 \}
\]

which under our CES form yields -

\[
P_{x_i} = e_i(P_{x_i}) = \left( \sum_j \beta_{ij} P_{x_j}^{1-\sigma_i} \right)^{\frac{1}{\sigma_i}}
\]

(12)

here \( P_{x_j} \) is the price of variety \( j \) of product \( i \).

Then via Shephard’s Lemma we get the following conditional demand function -

\[
X_{ij} = X_i \frac{\partial e_i}{\partial P_{x_j}} = \beta_{ij}^\sigma X_i \left( \frac{P_{x_j}}{P_{x_i}} \right)^{-\sigma_i}
\]

(13)

where \( \sigma_i = AE \) (the elasticity of substitution in the \( i^{th} \) market) = \( \frac{1}{1+\rho_i} \).

The market demand for a product is a function of income and prices in its own and other markets:

\[
X_i = X_i(M, P_{x_1}, P_{x_2})
\]

(14)

Now \( P_{x_i} \) has a property that -

\[
\frac{dP_{x_i}}{P_{x_i}} = \sum_{k \neq i} S_{ik} \frac{dP_{x_k}}{P_{x_k}} \quad \text{where} \quad S_{ik} = \frac{P_{x_k} X_{ik}}{P_{x_i} X_i}
\]

(15)

Differentiating (13) and dividing through by \( X_{ij} \) and after some manipulation -

\[
\frac{dX_{ij}}{X_{ij}} = \frac{dX_i}{X_i} - \sigma_i \left( \frac{dP_{x_j}}{P_{x_j}} - \frac{dP_{x_i}}{P_{x_i}} \right)
\]

(16)

Differentiate (14), substitute into (16) and use (15) to get –

\[
\frac{dX_{ij}}{X_{ij}} = \varepsilon_i \frac{dD}{D} - [(1-S_{ij})\sigma_i + S_{ij} \eta_{x_i}] \frac{dP_{x_j}}{P_{x_j}} + \sum_{k \neq i} [S_{ik} \sigma_i - S_{ik} \eta_{x_k}] \frac{dP_{x_k}}{P_{x_k}} + \sum_{j \neq i} \eta_{x_j} \frac{dP_{x_j}}{P_{x_j}}
\]

(17)
This is a general formulation of (17), in our model the third and fourth terms on the RHS do not need a summation since we have only two skill/technology/capital intensive products, and because we will assume only two varieties of each product.

Differentiate (14) and (13), and use (15) to get the percentage change in demand for $X_{ij}$ in value terms –

$$
\frac{d(P_{x_{ij}} X_{ij})}{P_{x_{ij}} X_{ij}} = \epsilon_i \frac{dM}{M} - [(1 - S_{ij})(\sigma_i - 1) + S_j (\eta_{X_i} - 1)] \frac{dP_{x_{ij}}}{P_{x_{ij}}} \\
+ \sum_{k \neq i} [S_{ik} (\sigma_i - 1) - S_{ik} (\eta_{X_i} - 1)] \frac{dP_{x_k}}{P_{x_k}} + \sum_{f \neq i} \eta_{X_i X_f} \frac{dP_{x_f}}{P_{x_f}} \tag{18}
$$

In (17) and (18) $\epsilon_i$ is the income elasticity of demand for product $i$, $\eta_{X_i}$ is the direct price elasticity of demand for product $i$, $\eta_{X_i X_f}$ is the cross elasticity of demand for product $i$ with respect to changes in the price of product $f$. The coefficient of $\frac{dP_{x_{ij}}}{P_{x_{ij}}}$ is the direct price elasticity of demand for $X_{ij}$ in (17) and the price elasticity of demand for $X_{ij}$ in value terms in (18). Similarly, coefficients of $\frac{dP_{x_k}}{P_{x_k}}$ in (17) and (18) are the cross price elasticity, of demand, and of demand in value terms, for $X_{ij}$ with respect to any competing variety in the market for product $X_i$.

Feenstra, et al. (2012) separate the AE into two categories for a given product market: the macro AE between home and imported varieties, and the micro AE among imported varieties. In this paper we focus on the macro elasticity and therefore we collapse demand for all imported varieties from all foreign countries into a single variety $k$ of product $i$. Then $X_{ik}$ is the imported variety and the only other variety, $X_{ij}$, is domestically produced.

II.3 Reflecting only the Supply Side Forces on Variety Elasticities
In this section we allow AE to vary with technology, to isolate the impact of supply side forces of our model (Section II.1 above). At the same time, we hold demand elasticity of product $i$ constant. This theoretical set-up yields a framework to analyse the Asian dominance of the lithium-ion battery market, an example that we presented in the introduction to this paper. The Asian governments’ support have helped producers to leapfrog ahead in technology to where $\sigma = \sigma_H$, this is in some cases well beyond the level that would be predicted by their actual income levels (Onstad, et.al., 2018). Government support has also helped the producers garner a large market share, thus $S_y$ close to 1. The theoretical set-up mimics this leapfrogging by allowing technology to improve in a particular sector, even when demand side forces remain constant in the short run because the economy as a whole remains at a lower level of development.

We have assumed, ceteris paribus, that the AE between varieties in the market for product $i$ is $\sigma_y$ when technical capacity is high, making it easy for consumers to substitute between imports and domestic varieties of product $i$. It is $\sigma_L$ for low technological capacity $\sigma_L < \sigma_H$. Ordinarily, we would expect $\sigma_L$ and $\sigma_H$ to exceed unity, thus an improvement in competitiveness would yield an increased market share. But our model places no such restriction, which fits well with our intuitive framework where a reduction in price may not help customers adopt a particular variety if production technology differences make the varieties poor substitutes.

Using (2) in (18), the own price elasticity of variety $j$ of product $i$ -

$$\eta_{x_j} = (1 - S_{ij})\frac{m_{\text{min}}(\sigma_L \sigma_H)}{\sigma_L - 1} + S_y(\eta_{X_y} - 1).$$

(19)

Again using (2) in (18) the cross price elasticity of variety $j$ with respect to change in the price of variety $k$ of product $i$ -

$$\eta_{x_jx_k} = S_{ik}\frac{m_{\text{min}}(\sigma_L \sigma_H)}{\sigma_L - 1} - S_{ik}(\eta_{X_k} - 1).$$

(20)

II.4 Reflecting only the Demand Side Forces on Variety Elasticities
Here we allow price elasticity of product \( i \) to vary with income (the demand side forces from Section II.2), but hold AE constant. A real world situation that corresponds to this theoretical set-up is the example of the Indian consumer automobile market that we presented in the introduction. Due to the lack of competition, in the closed Indian economy, technology remained stagnant in that example, say \( \sigma = \sigma_L \). In addition the market share of the domestic automobiles was very high due to trade protection, say \( S_y = 1 \). In the meantime, however, India’s per capita GDP was rising therefore leading to steadier demand for technology intensive products such as automobiles. This example therefore corresponds to the theoretical set-up in this section where we assume that only demand side forces operate, while technology does not improve.

Substitute the price elasticity of demand for product \( i \) from equation (10) and use the definition of \( m_0 \) from equation (11) to write the own price elasticity in equation (18) of variety \( j \) of product \( i \) in the following form -

\[
\eta_{x_i} = (1-S_y)\left(\sigma_i - 1\right) + S_y \left(\frac{m - \bar{x}_2 p_{s_2}}{m - m_0} - 1\right).
\]  

Similarly we obtain the cross price elasticity of variety \( j \) with respect to change in the price of variety \( k \) of product \( i \) -

\[
\eta_{x_{i,k}} = S_{yk} \left(\sigma_i - 1\right) - S_{ik} \left(\frac{m - \bar{x}_2 p_{s_2}}{m - m_0} - 1\right).
\]  

II.5 Bringing Together the Supply and Demand Side Forces

In this section we allow both demand and supply side factors to operate, so, AE varies with technology and price elasticity of product \( i \) varies with per capita income. Thus accounting for both the supply and demand side impacts together gives us the following expression for the own price elasticity of a variety -

\[
\eta_{x_j} = (1-S_y)\left[\eta L \left(\frac{m - \bar{x}_2 p_{s_2}}{m - m_0} - 1\right) \right] + S_y \left(\frac{m - \bar{x}_2 p_{s_2}}{m - m_0} - 1\right) \]  

Similarly, the cross price elasticity of demand -

$$\eta_{x_i x_k} = S_h \left[ \sigma_L \frac{m}{\prod_{j}^{\infty} \frac{m}{\sigma}} - 1 \right] - S_m \left[ \frac{m - \bar{x}_2 p_{x_2}}{m - \bar{m}} - 1 \right]$$  \hspace{1cm} (24)$$

We obtain the change in own price elasticity $\xi$ as a result of a change in per capita income by differentiating equation (23) with respect to $m$–

$$\frac{d \eta_{x_i}}{dm} = (1 - S_y) \left[ \sigma_L \ln \left( \frac{\sigma_H}{m} \left( \frac{\sigma_H}{\sigma_L} \right)^{\bar{m}} \right) - \frac{S_m (1 - \xi) \bar{m}}{(m - \bar{m})^2} \right]$$  \hspace{1cm} (25)$$

Here $\theta$ is the proportion of income spent on $x_2$.

Similarly we obtain the change in cross price elasticity as a result of a change in per capita income by differentiating equation (24) with respect to $m$:

$$\frac{d \eta_{x_i x_k}}{dm} = S_h \left[ \sigma_L \ln \left( \frac{\sigma_H}{m} \left( \frac{\sigma_H}{\sigma_L} \right)^{\bar{m}} \right) \right] + \frac{S_m (1 - \xi) \bar{m}}{(m - \bar{m})^2}$$  \hspace{1cm} (26)$$

(25) and (26) present the trajectory of trade competitiveness of a country by quantifying how market demand elasticity evolve with economic development.

II.6 Interpreting the Model

It is useful to review the salient mechanisms at work, in the model –

1) Role of development. Development is measured by per-capita income and the level of technology.

(a) From (10), own price elasticity for the skill/capital/technology intensive product is declining in development. Intuitively, this means more developed economies will have a steadier demand for these products. In the Indian consumer automobile market, this was the growing incomes of the consumers, rising $m$, in the decades of the 60’s, 70’s and 80’s. This led to a robust demand for cars.
(b) From (2) AE will converge towards the high level, $\sigma_H$, as development tends towards the minimum needed to ensure the economy has access to world class technology. Intuitively, the economy’s development leads to progress towards the world technology frontier. Examples of this would be most developed countries of the world where income, $m$, rise and technological progress, $T$, have occurred simultaneously. It is however also possible that technology leapfrogs to a higher level than predicted by the country’s development, i.e., $\sigma = \sigma_H$ even when $m < T_{\text{min}}$. This is the case among some Asian producers of lithium ion batteries due to government support for technology acquisition. The leapfrogging Asian lithium-ion battery manufacturers are analyzed in the model by setting the supply side at starting at $\sigma_H$ while the demand side is held constant, where $m < T_{\text{min}}$. Indian automobile manufacturers that experienced stagnant technology are analyzed in the model as being in a market where $\sigma$ is constant at say $\sigma_L$, and no longer a function of income, while the demand side continues to evolve as $m$ rises.

2) Role of the market shares of varieties of a product. The larger the initial market share of a product variety, the smaller will be the reaction of its demand to both its own and other variety’s price changes.

(a) In (17) the coefficient of the price change of variety $j$ indicates that, as long as $\sigma_i > \eta_{X_i}$, the own price elasticity is smaller for a variety with a larger market share.

(b) Also in (17) the coefficient of the price change of the other variety, $k$, of the product indicates that as long as $\sigma_i > \eta_{X_i}$, the cross price elasticity is also smaller for variety $j$ with larger market share.
In our simulations, presented in Section III below, incomes range from the 25\textsuperscript{th} to 75\textsuperscript{th} percentile. In this range \( \eta \) varies between .05 and .16 while \( \sigma \) varies between 0.5 and 3.1. Therefore the condition in 2) (a) and 2) (b), listed above, holds.

III. NUMERICAL SIMULATION

We simulate the response of the analytical results presented, in Section II.3, II.4 and II.5 above, to changes in per capita income. Our simulations study an area of real world importance. We’ve presented a number of real world examples throughout this paper (see the introduction and Sections II.1). Our aim is to determine how the level of a country’s development impacts the demand elasticity of the variety it produces of a given product. In addition we analyse how this impact changes as the economy develops and converges towards the world technology frontier. Simulations indicate that the answer depends crucially on the domestic demand for these skill/technology/capital intensive products, the distance of the country from the world technology frontier, as well as the initial market share of the country’s variety.

Table 2 presents the range of parameter values selected and explanations for each. The parameter value for market share for the imported variety ranges from 0 to 100 percent of the total market for a product. Our assumption, without any loss of generality under the Armington (1969) model, is that there are two varieties of each product: imported and domestically produced. Given the strong positive relation between per capita income and technological sophistication (Schott 2004, Balassa 1979, Romalis 2004, Hallak 2006, Barro 1991, Linder 1961), in our model simulations the percentile per capita income represents the technology index, \( T \), as well. The per capita incomes range from the 25\textsuperscript{th} to the 75\textsuperscript{th} percentile. The 25\textsuperscript{th} percentile is, \( m_0 \), the minimum income required for the country to start consumption of the skill/capital/technology intensive product. The 75\textsuperscript{th} percentile is, \( T_{\text{Min}} \), the level of development at which the country has attained the world technology frontier and the AE is therefore high = \( \sigma_H \). These percentiles correspond closely
to those used in the literature (E.g. Schott, 2004) and used by data sources that categorize countries into low, middle, and high income categories (E.g. the World Bank’s Open Access Database).

Furthermore, in our numerical simulations, the elasticity of substitution ranges from .5 ($\sigma_L$) to 3.1 ($\sigma_H$), this range has been estimated in the literature (Feenstra et al. (2012) and Broda and Weinstein (2008)).

The figures from 1 to 6 presents the simulations as the level of a country’s development, measured by its per-capita income, increases from the 25th to the 75th percentile. The initial market share of the country’s domestic variety is allowed to vary from 0 to 100% in each case.

III.1 Simulation results when both the Supply and Demand Side Forces work together

Own price elasticity -

Figure 1 presents the simulations of equation (25) as the level of a country’s development changes. We see that initially, at low levels of income, development leads to falling elasticity, however beyond a threshold development leads to rising elasticity of the domestic variety. Therefore the demand side forces come into play early in the development process, with steadier demand for skill/capital/technology. At this stage due to technological differences, the two varieties of the product are very different from each other. In the later stages, as the economy closes in towards the world technology frontier, elasticity rises as domestic versus foreign varieties become comparable in quality and thus easier to substitute.

The general impact of market shares that we observe from Figure 1 is that the larger the initial market share of a variety, the higher the threshold at which its elasticity will rise with income. Therefore the demand side forces seem to hold sway for longer for the product variety with larger market share. Its elasticity continues to decline for a longer range of the development cycle as well. It is only when the economy reaches very close to the world technology frontier that the
variety finally experiences a rise in elasticity. Even then, the rise is much less than in a variety with a smaller market share.

This numerical exercise can be applied to studying the impact of government policy of helping domestic producers gain market dominating shares. Specifically we can cover the spectrum of possibilities with a country starting to the left of $T_{\text{min}}$ and another to the right of $T_{\text{min}}$. The Indian automobile manufacturing example is illustrative of a case where technology is below the world frontier technology threshold of $T_{\text{min}}$. The Asian electric car battery example is illustrative of a case where technology is at or above this threshold.

Consider the example of the Indian consumer automobile market first. Throughout the decades of the 60s, 70s and 80s the Padmini and Ambassador did well as incomes grew (Sumantran, 1993). These were demand side forces. However due to lack of innovation in the protectionist economy, the supply side forces were never unleashed. Thus when trade was finally opened up, domestic car varieties were entirely obsolete and could not compete with the foreigners. Note that while the model does not explicitly discuss trade policy, trade liberalization can be clearly analyzed within the model framework by a drop in the market share of the domestic variety, $S_{ij}$. With falling market share (represented, for example, by shift of the graph from a 1.0 market share of variety $j$, the domestic variety, to a 0.1 share) the demand is more vulnerable to price changes. This can be seen by the fact that elasticity rises in figure 1. This demonstrates the poor fate of the domestic car manufacturing in India with trade liberalization.

A clear contrast with the Indian automobile example presented above, is the case of Asian dominance of the electric car battery market is instructive here. The large initial market share, say $S_{ij} = 1$, helps keep demand for Asian-made batteries strong. While in this case all countries involved, European as well as Asian are at the world technology frontier, i.e., $T \geq T_{\text{min}}$ and $\sigma = \sigma_H$ for all producers. Here the prediction of the model where closeness to the world technology frontier
leads to rising elasticity might only come about with the next generation of “solid state” batteries puts everyone on a more level playing field in terms of market shares. The absolute market dominance graph with the 1.0 market share of the Asian variety seems to be the closest to reality for this market.

**Cross price elasticity**

Figure 2 presents the simulations of equation (26). Cross price elasticity rises with income, at all levels of income. The rise is higher at lower levels of income. Higher market share of the other variety (whose price is changing) creates a higher rise in cross price elasticity of variety $j$.

An instructive example is the relocation of Asian battery manufacturers to Europe, which is projected to bring down their prices due to saving transport cost (Onstad, et al., 2018). This price change is not expected to change their market share by much, it is already very high. But this price change is expected to sweep out all European competition from the market reducing their market share dramatically.

**III.2 Simulation results when only Supply or Demand Side Forces work**

**Own elasticity: only demand side forces operating**

Figure 3 represents this case. The variety elasticity declines with income at all levels of income. Decline is more with higher market shares, as demand becomes increasingly steadier. But the greatest decline is at the point where the economy reaches the level of income where it starts consuming skill/capital/technology intensive products.

**Own elasticity: only supply side forces operating**

Figure 4 represents this case. The variety elasticity increases with income at all levels of income. The increase is higher at higher income, as rising income makes technology improve and varieties become easier to substitute for one another, so a small change in the price of one variety
would result in large change in quantity demanded. This change is dampened only by higher market share of the variety. The higher market share advantage remains, while those with lower market shares find themselves becoming more vulnerable to price changes.

**Cross elasticity: only demand side forces operating**

Refer to figure 5 for this case. The change in cross price elasticity of the variety as the income level varies, is positive throughout. It is higher at lower income levels, the greatest impact is at \( m_o \). As usual, the higher the market share of the variety whose price changes (variety \( k \)), the greater is the impact on variety \( j \).

**Cross elasticity: only supply side forces operating**

Refer to figure 6 for this case. The change in cross price elasticity for the variety is again positive throughout the spectrum of per capita income. The supply side impact increases as technology converges to \( T_{Min} \). As in the previous case, the market share of the other variety (\( k \)) is linked positively with the cross elasticity of variety \( j \).

### III.3 General Conclusions

Across both own and cross price elasticity of a variety of the product we see that: 1) market share is negatively related to the elasticity for the variety, 2) the demand side impact is strongest at \( m_o \), and 3) the supply side impact is strongest at \( T_{Min} \). The knowledge of the threshold points that 2) and 3) represent, provides insights into the theory behind the Armington approach widely used in international trade modelling. In addition, it can be useful to both policy makers and competing firms in these markets.

### III.4 The Scope of our Model

Our model’s purpose is to analyze how trade competitiveness, in particular the ease of substitution between domestic production and imports, changes over the course of development.
However the process of development and therefore the level of income, are determined outside the model. This is because in the real world, the process of development has many contributing factors including economic fundamentals, historical/cultural aspects, and government policy. Thus the structural determination of the level of income and AE is beyond the scope of the current model.

IV. CONCLUSION

The theoretical contribution of the model in this paper is to enrich, the widely used, Armington approach (in for example CGE models) by accounting for the supply and demand side implications of technological and economic development. The model yields a theoretical framework where technology, market shares, and consumers’ demand for technologically advanced products are closely linked. Our simulations present the implications for the competitiveness of the domestic variety over the course of economic development.

This framework also has widespread policy applications in international trade. On trade policy, it informs on the evolution of trade competitiveness of nations as they develop. This is useful in a time of rising trade protectionism evidenced by failures of the Doha trade round at the WTO and the looming trade war between the US and China. Furthermore, the simulations give us an interesting insight into the hoped-for versus the realized impact, of policy intervention. We have seen that government policy initiatives might aim to increase market share and/or accelerate technological progress. We see from our model that both the demand and supply side can help spur demand for domestically produced skill/capital/technology intensive product’s variety produced at home. However if the government’s policy is only nurturing the demand side forces, say through trade protection, the domestically produced variety might only benefit while the protection lasts. When the economy is opened up to foreign competition, e.g. India in the 1990s, the domestic producers will lose out in the absence of supply side technology advances.

Strategic government intervention that targets technological progress, and results in market share gain based on that solid foundation, might lay groundwork for long term strength of domestic
producers the example of the Asian producers of electric car batteries comes to mind. This might be
the difference between the success of government policy in promoting domestic companies in the
rest of Asia, versus the failure in pre-1990s India.

While this discussion appears to hold a somewhat cautionary note for governments that
promote inward looking policies coupled with trade protection, in reality, the level of protection
countries choose might not be optimal for several reasons, some unrelated to economic
fundamentals. For example, firms can lobby governments to implement suboptimal levels of
protection, and this is true for both developing and developed countries. For example Moore (1992)
and Aquilante (2018) provide evidence on how the use of antidumping measures in the U.S. is
linked to the geography of employment across states, despite the very peculiar system the U.S. has
in place to for antidumping investigations which is meant to shield the use of antidumping from
political influence.

References
Alaouze, C. M. (1977). Estimates of the elasticity of substitution between imported and domestically produced goods
classified at the input-output level of aggregation (No. 0-13). Victoria University, Centre of Policy Studies/IMPACT
Centre.

159-178.


Economics and Statistics 61(2): 259-266

Perspectives, 30(1), 95-116.

407-443.


Schott, P. K. (2002). Moving up and moving out: Us product-level exports and competition from low wage countries. Yale School of Management *mimeo*.


Table 1 - AE estimates from the literature with patent share to measure technology development

<table>
<thead>
<tr>
<th>Data (trading countries, sectors)</th>
<th>Years</th>
<th>Patents filed by country in time period (% of world total)</th>
<th>Estimated Armington Elasticity (AE), average and range</th>
<th>Publication</th>
</tr>
</thead>
<tbody>
<tr>
<td>AE estimates between domestic &amp; imported varieties</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>46 imports (Australia; 4-digit ASIC)</td>
<td>1968 - 75</td>
<td>0.27 %</td>
<td>0.6-4.4</td>
<td>Alaouze (1977)</td>
</tr>
<tr>
<td>26 sectors (Portugal)</td>
<td>1962 - 78</td>
<td>0.02 %</td>
<td>0.79 (range is -0.3-2.3)</td>
<td>Corado &amp; de Melo (1986)</td>
</tr>
<tr>
<td>22 sectors (U.S. imports of mining &amp; manufacturing from Canada, Mexico and ROW)</td>
<td>1980 - 88</td>
<td>14 %</td>
<td>0.1-1.5</td>
<td>Shiells &amp; Reinert (1993)</td>
</tr>
<tr>
<td>30 imported commodities (Philippines)</td>
<td>Mid 1970- late 1980</td>
<td>0.02%</td>
<td>1 (range is 0.2-4.1)</td>
<td>Kapuscinski &amp; Warr (1999)</td>
</tr>
<tr>
<td>151 sectors (U.S. manufacturing 3-digit SIC)</td>
<td>1980 - 88</td>
<td>14 %</td>
<td>0.81</td>
<td>Blognigen &amp; Wilson (1999)</td>
</tr>
<tr>
<td>309 sectors (U.S.; 4-digit SIC)</td>
<td>1989 - 95</td>
<td>16 %</td>
<td>0.95 (short run) 1.55 (long run)</td>
<td>Gallaway et al. (2003)</td>
</tr>
<tr>
<td>Wood, and pulp/paper products (U.S.)</td>
<td>1961 - 2002</td>
<td>18 %</td>
<td>0.15 (short run) 0.62 (long run)</td>
<td>Gan (2005)</td>
</tr>
<tr>
<td>11 sectors (France)</td>
<td>1970 - 97</td>
<td>2 %</td>
<td>0.84 in 1976-91 0.21 in 1982-97 (result of tariff reduction)</td>
<td>Welsch (2006)</td>
</tr>
<tr>
<td>AE among imported varieties</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26 sectors (Portugal)</td>
<td>1962 - 78</td>
<td>0.02 %</td>
<td>1.5 (range -0.6 - 3.3)</td>
<td>Corado &amp; de Melo (1986)</td>
</tr>
<tr>
<td>163 commodities (U.S. mining &amp; manufacturing; 3-digit SIC)</td>
<td>1980 - 88</td>
<td>14 %</td>
<td>0.91 (range 0.14 - 3.49)</td>
<td>Reinert &amp; Roland Holst (1992)</td>
</tr>
<tr>
<td>22 sectors (U.S. imports of mining &amp; manufacturing from Canada, Mexico and ROW)</td>
<td>1980 - 88</td>
<td>14 %</td>
<td>0.039 - 2.97</td>
<td>Shiells &amp; Reinert (1993)</td>
</tr>
<tr>
<td>57 goods bilateral trade (U.S., New Zealand, Argentina, Brazil, Chile, Paraguay)</td>
<td>1992</td>
<td>15 % (U.S.); 0.2 % (New Zealand); 0.08 % (Argentina); 0.3 % (Brazil); 0.03 % (Chile); 0.003 % (Paraguay)</td>
<td>5.6 (range 3 - 8, goods within Machinery have the highest estimates, impact of tariff changes)</td>
<td>Hummels (2001)</td>
</tr>
<tr>
<td>Bilateral trade (16 OECD countries)</td>
<td>1958 - 60 &amp; 1986 - 88</td>
<td>95 % (total of 16 countries)</td>
<td>6.43</td>
<td>Baier &amp; Berstrand (2001)</td>
</tr>
<tr>
<td>Bilateral trade (19 OECD countries)</td>
<td>1990</td>
<td>95 % (total of 19 countries)</td>
<td>8.23</td>
<td>Eaton &amp; Kortum (2002)</td>
</tr>
<tr>
<td>4655 commodities (U.S., Canada, Mexico, HS-6)</td>
<td>1989 - 99</td>
<td>17 % (U.S.) 0.5 % (Canada) 0.09 % (Mexico)</td>
<td>6.2 - 10.9 (impact of tariff changes)</td>
<td>Romalis (2002)</td>
</tr>
<tr>
<td>Rice (Indonesia)</td>
<td>1985 - 2003</td>
<td>0.01 %</td>
<td>2.6 - 4.1</td>
<td>Warr (2005)</td>
</tr>
</tbody>
</table>
### Table 2 – Parameter Values for Numerical Simulation of Equations (25) and (26)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Values</th>
<th>Justification of Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{iq}$</td>
<td>Market-share of variety $q$ in the market for product $i$</td>
<td>0-100 percent of the market</td>
<td>Entire range of possible values is included.</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>Assumed low value of the Armington elasticity, when the technological capacity and per capita income are low.</td>
<td>0.5</td>
<td>The lowest value of the spectrum of values derived for Armington elasticity in Feenstra et al. (2012)</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>Assumed high value of the Armington elasticity, when the technological capacity and per capita income are high.</td>
<td>3.1</td>
<td>The value of elasticity of substitution obtained in Broda and Weinstein (2008) and this is comparable to the highest value of Armington elasticity obtained in Feenstra et al. (2012)</td>
</tr>
<tr>
<td>$m_T = T_{Min}$</td>
<td>The per capita income percentile and the technology index, at which the economy’s technology level is high enough to produce good quality import substitutes such that $\sigma_H$ prevails.</td>
<td>75th percentile of per capita income</td>
<td>These match with the World Bank’s classification of high and low income countries</td>
</tr>
<tr>
<td>$m_0$</td>
<td>The income at which economy’s technology level is so low as to produce import substitutes such that $\sigma_L$ prevails</td>
<td>25th percentile of income</td>
<td>We assume that on average half of income is used to purchase the $x$ products and half to purchase the $y$. And a quarter each goes to $x_1$ and $x_2$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>The share of income spent on product $x_2$</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

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Figure 1: Change in Own Price Elasticity of Variety \( j \) as Income Changes

Both demand and supply side forces operating

\[
\frac{d \eta_{x_{ij}}}{d m}
\]
Figure 2: Change in Cross Price Elasticity of Variety $j$ as Income Changes

Both demand and supply side forces operating
Figure 3: Change in Own Price Elasticity of Variety $j$ as Income Changes

*Only demand side forces operating*
Figure 4: Change in Own Price Elasticity of Variety \( j \) as Income Changes

*Only supply side forces operating*
Figure 5: Change in Cross Price Elasticity of Variety \( j \) as Income Changes

*Only demand side forces operating*
Figure 6: Change in Cross Price Elasticity of Variety $j$ as Income Changes

Only supply side forces operating

\[
\frac{d q_{xij}^x}{dm} = \text{market share of the other variety } k
\]