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The difference between noise and a musical sound is the regularity of vibrations. Sound waves are a succession of impulses started by a vibrating body and carried by air and sensed by our ears. The air is alternately compressed and expanded caused by a string or reed which makes the air vibrate and the pressure wave travels outwards. The frequency of the vibration is the number of cycles per second, measured in Hertz (Hz), and determines the note or pitch. The more vibrations per second, the higher the pitch.

The musical instruments used in an orchestra are largely played by either drawing a bow across a string or by plucking the string with a finger, by blowing through a tube or by hitting surfaced such as stretched skins. Grouped into string, wind and percussion, the pitch of the different instruments depends on the length of string or tube, or the size of skin. The stringed instruments in an orchestra include violins, violas, cellos and double basses. In the case of the smallest and highest sounding stringed instrument, the violin, there are four tensioned strings over a resonant wooden box that is pieced by two f-shaped sound holes (Figure 1).

When a string stretched between two points is plucked it vibrates. A transverse wave is created since the vibrations are from side to side. That is, the oscillations are perpendicular to the direction of the wave or path of propagation. The speed of the wave and the wavelength determine the frequency of the sound produced. The frequency is also dependent on the physical characteristics of the strings. For instruments which have a number of strings all of the same length, requires them to be of different thicknesses to create a range of difference frequencies or notes. Strings may be made from similar materials and each therefore has a different linear density. This is, they have a different mass per length of string.

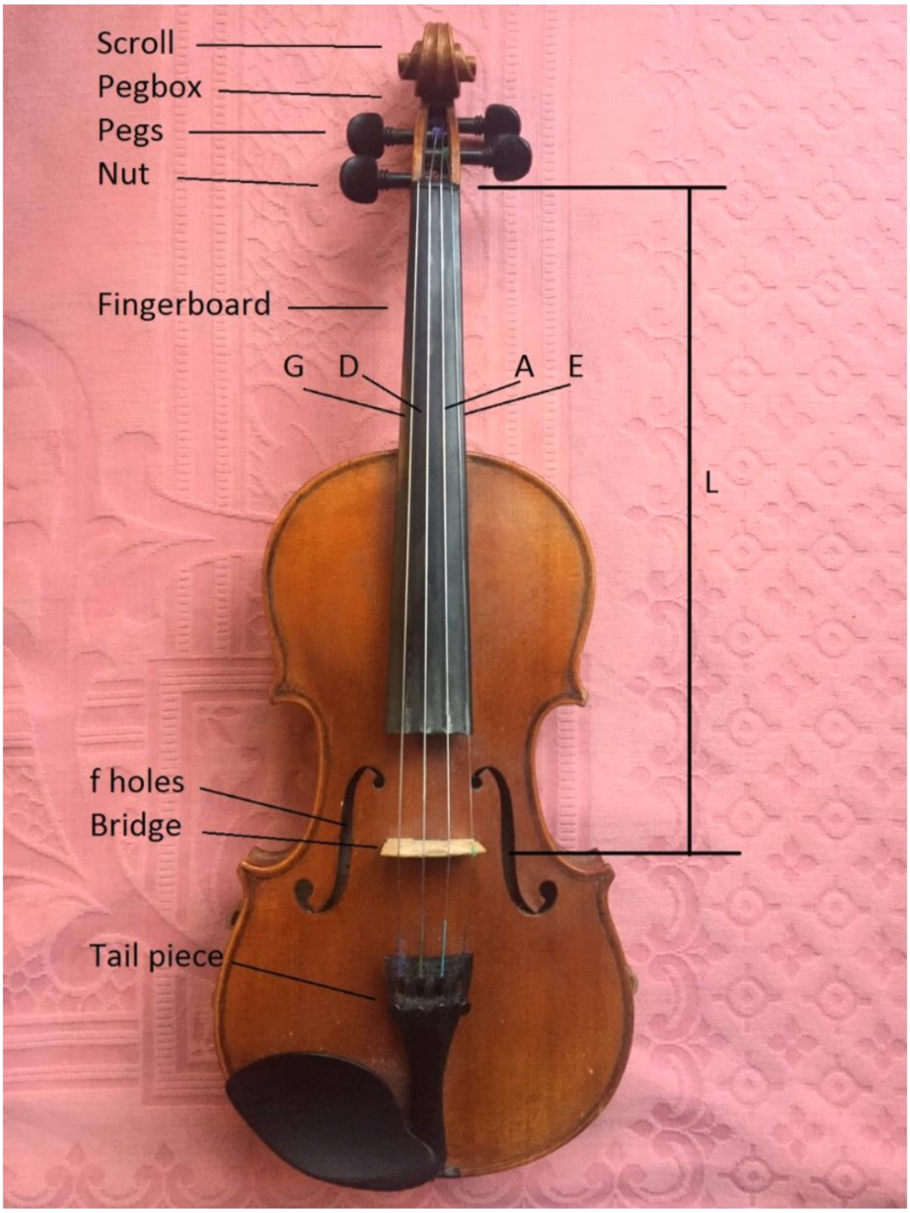


Figure 1 Features of a violin

The strings are attached at one end by a tail piece and tensioned by pegs within the pegbox. The violin placed under the player's chin and is played by drawing a bow that is held in the right hand across the strings causing them to vibrate. Other than those of the open strings, the notes are played by pressing the string with the fingers of the left hand onto the fingerboard thereby effectively shortening the string to produce higher frequencies or notes. The strings are sometimes plucked, called pizzicato.

The frequency of vibration, f (s^{-1}), of a violin string depends on the length of the string, L (m) between the nodes, the tension, T (kg ms^{-2}), in the string and the linear density, m (kg m^{-1}). The relationship between these variables can be determined from:

$$f = kL^a T^b m^c$$

The fundamental dimensions of length, mass and time are therefore

$$[T^{-1}] = k[L]^a [MLT^{-2}]^b [ML^{-1}]^c$$

Equating the dimensions:

$$\begin{aligned} M: &\quad 0 = b + c \\ L: &\quad 0 = a + b - c \\ T: &\quad -1 = -2b \end{aligned}$$

That is, the exponents are $a = 1$, $b = \frac{1}{2}$ and $c = -\frac{1}{2}$. The relationship between the variables is therefore:

$$f = kL^{-1}T^{1/2}m^{-1/2}$$

or

$$f = \frac{k}{L}\sqrt{\frac{T}{m}}$$

The constant k has a value of 0.5 (Mussard, 1993). This means that for a fixed length of string, the different frequencies for each string is dependent on the linear density. Each is "tuned" by changing the tensions in each of the strings using the pegs. The standard notes of the four strings are G, D, A and E and are tuned to vibrate at the fundamental frequencies 196 Hz, 293.76 Hz, 440 Hz, 659.3 Hz, respectively. That is, the G string is the thickest with the highest linear density and the E string the thinnest. To play notes other than these fundamental notes, the effective length of the strings needs to be reduced by pressing down on the strings with the fingers to effectively shorten it. To ensure all the instruments are in tune together, an orchestra will begin a performance by ensuring all the players are first in tune to A (440 Hz).

When a string is caused to vibrate, the point of greatest movement of the string is called the anti-node and the point of no movement is called the node. Touching a vibrating string at exactly the centre creates a node with two anti-modes. The string will therefore play the same note but sound a second harmonic. The two nodes of a violin string are located at the bridge and the top of the fingerboard, called the nut. Figure 2 represents a simplistic wave form and harmonic. The actual motion of vibrating violin strings, however, was first predicted by the German physicist, Hermann Ludwig Ferdinand von Helmholtz (1821 -1894).

In what is known a Helmholtz motion, the string vibrates with a V-shape wave which travels back and forth along the string.

The human ear is able to judge the relationship between certain series of notes. A ratio of a series of notes is called a scale. Musical notes are separated by a factor of $2^{1/12}$ and an octave spans a factor of two in frequency with twelve notes per octave shown in Table 1 (Baker, 2010). The black notes are the sharps and flats found on a piano.

Table 1 Frequency of Musical Notes

Notes	Piano key	Relative frequency as a power of 2	Frequency (5 Octaves) Hz				
A		1	55.00	110.00	220.00	440.00	880.00
B♭/A#	black	$2^{1/12}$	58.27	116.54	233.08	466.16	932.32
B		$2^{2/12}$	61.74	123.48	246.96	493.92	987.84
C		$2^{3/12}$	65.41	130.82	261.64	523.28	1046.56
D♭/C#	black	$2^{4/12}$	69.30	138.60	277.20	554.40	1108.80
D		$2^{5/12}$	73.42	146.84	293.68	587.36	1174.72
E♭/D#	black	$2^{6/12}$	77.78	155.56	311.12	622.24	1244.48
E		$2^{7/12}$	82.41	164.82	329.64	659.28	1318.56
F		$2^{8/12}$	87.31	174.62	349.24	698.48	1396.96
G♭/F#	black	$2^{9/12}$	92.50	185.00	370.00	740.00	1480.00
G		$2^{10/12}$	98.00	196.00	392.00	784.00	1568.00
A♭/G#	black	$2^{11/12}$	103.83	207.66	415.32	830.64	1661.28
A		2	110.00	220.00	440.00	880.00	1720.00

By starting at any note, the frequency to other notes can be calculated from:

$$f = note \times 2^{N/12}$$

where N is the number of notes away from the starting note. This can be positive, negative or zero. For example, starting at A (440.00 Hz), the frequency to the next string on the violin, which is E, is:

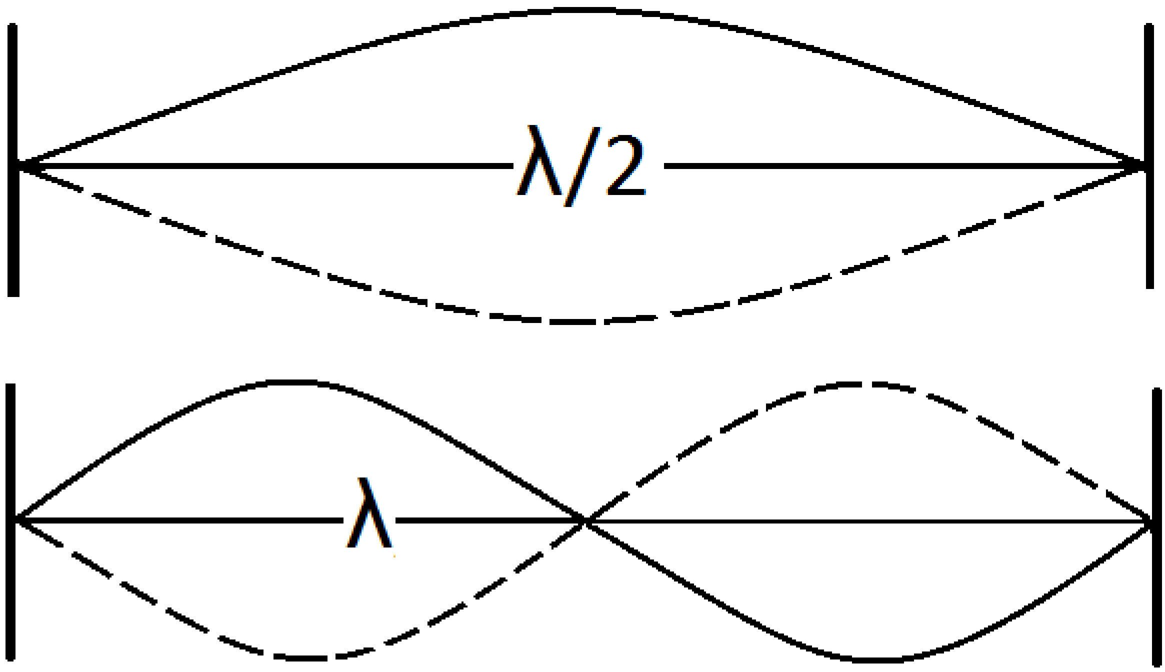
$$440 \times 2^{7/12} = 659.3\ Hz$$

since E is seven notes above A. Likewise, the frequencies of the other two strings, D and G are, respectively:

$$440 \times 2^{-7/12} = 293.6\ Hz$$
$$440 \times 2^{-14/12} = 196.0\ Hz$$

The speed of a wave is calculated from the product of the frequency and wavelength. In the case of a plucked string, the wavelength, λ, is half the distance between the nodes (Figure 1).

Figure 2 Wavelength of Vibrating Strings



Investigations with the sounds of vibrating strings of musical instruments are fun to do. It is worth working with musicians to help make the sounds such as the players in a school orchestra. With different stringed instruments you might try to work out how to produce different notes (frequencies) on different strings. Try to find harmonics or compare the sounds and notes produced by different instruments.

You can also investigate what, for example, are the hairs of a bow made from and what is the purpose of rosin? What are frets and why does a violin not have any? What is the purpose of the f holes and what is the highest note that can be played on a violin? What causes a violin to go "out of tune"? How does the sound change if a string is bowed close to the bridge (*sul ponticello*) or further away (*sul tasto*).

From a physics point of view, you can also calculate the tension of a string needed to create a different frequency. You may also consider that given the linear densities of strings, what is the overall tension on the neck of a stringed instrument? What is the effect of air pressure and what would happen if played at the top of a mountain or under water? What is the reduction in length of a string that is needed to produce a different note? In fact, this is what a player must do to play notes and musical tunes. Skilled players achieve this through practise and muscle memory. That is, they become very familiar with where to place their fingers without having to think about it. Try watching how players with different instruments are able to perform a musical tune.

Of course, the quality of the sound made by the player is dependent not only on the skill of the performer but also the craftsmanship of the instrument itself. The finest violins are said to have been originally produced by the exceptionally talented Amati family in Cremona in Italy over a period of three generations. Andrea Amati (c1505-1577) began a workshop with his sons Antonio and Girolamo, and then with Girolamo's son, Nicolò Amati (1596-1684). The tradition of Nicolò's workshop was then continued by others including the famous Antonio Stradivari (c1644-1737), who may have been his pupil, and produced a remarkable quality of violin. Apart from a few modifications introduced in the nineteenth century, the design of the violin has largely remained unchanged from those made by Stradivari.

References and useful websites

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