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A simulation-optimization approach for economic assessment of an imperfect serial production management under uncertainty

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Abstract
One of the critical quandaries in real-world production management systems is uncertainty in the product demand, which is closely associated to the product price. This uncertain behavior of demand affects production decisions in a system. In this context, this study develops a decision support framework for an imperfect serial production management system under uncertainty. The proposed framework is analyzed considering price-dependent uncertain demand and random defective rate in the system, thus considering both internal and external uncertainties of the system. Three distinct production models are formulated considering different types of uncertain relations among the product demand and market price. Closed form solutions of the decisions are obtained through hybrid of simulation and analytical optimization and are tested through an experimental case analysis. @RISK of Palisade Suite is utilized for simulation and to capture the uncertainty in defective proportion and demand elasticity parameters by creating several possibilities.

1. Introduction
Market demand of a product or service is price dependent in general. It is very common with easily substitutable products whose alternatives are available in the market at competitive prices [1]. Customers switch between different brands of the product frequently. Production decisions in the presence of dependent demand are complex because of inadequate data about demand. Generally, a manufacturing company can learn about the product demand by observing some signals from the market [2]. The first inventory model consisting of price-dependent demand was formulated by Whitin [3], who determined selling price and order size simultaneously. After him, several researchers have modified conventional production and inventory models considering various factors directly influencing market demand of the products.

Secondly, in practically existing manufacturing processes, defective quality products are also produced along with the perfect quality products because of process deterioration, raw-material imperfections, disruptions in the production system, and environmental factors [4]. These items are reworked, scrapped, or sold on discounted price as per product type and industry decisions. Researchers and practitioners are frequently enriching the production and inventory literature by providing decision models considering various types of imperfect production systems generating defective items in constant [5], random [6], and in fuzzy proportion [7]. Most of the researchers have focused on imperfect single-stage production management system and literature regarding imperfect two-stage and higher stages production management system is out of their focus yet.

Above considerations indicate that the serial production management systems usually face internal uncertainties in the form of random defective proportion and external uncertainties as random demand elasticity to the product price. Hence, the consideration of both uncertain categories is imperative for the formulation of imperfect multi-stage production system’s optimal production policy. In this context, Tayyab et al. [8] have recently provided a serial production management model considering internal uncertainty of the production management system. They incorporated a random defective proportion in model formulation where the defective proportion follows a famous beta distribution density function and showed that the batch size goes on increasing while moving toward the manufacturing setups with multi-production stages. Their research work well captured the internal uncertainty of the
serial production management system, but external uncertain factors were ignored.

The research at hand builds on the study presented by Tayyab et al. [8] by relaxing the condition that customer demand of the product is necessarily a constant number. We consider price-dependent uncertain demand of the product being produced by a multi-stage imperfect production management with random imperfect proportion. Product demand is considered as uncertainly determined by the retail price through three different relations: linear, quadratic, and exponential. Accordingly, three distinct production management models are formulated for each of the relation among product price and demand. The proposed models are solved by implementing analytical optimization approach. Closed form solutions of the maximum profit functions for the proposed cases are developed and are numerically tested through an experimental case study. Uncertainty in the demand elasticity parameters and imperfect proportion at any production stage of the production system cannot be handled solely through expectation approach of probability theory, and thus this uncertainty is captured through Monte Carlo simulation using @RISK simulation add-in for Microsoft Excel which is scarce in production and inventory management literature. Then 100,000 different scenarios of the uncertain conditions are generated in the Monte Carlo simulation to obtain robust decisions for the proposed models. Sensitivity analysis for the proposed production models is conducted to analyze the variation in optimal profit for various levels of uncertainty to obtain key managerial insights.

The rest of the study is structured as follows. Section 2 studies milestones and limitations of the related production management literature. Four different production models are formulated in Section 3. Further, numerical experiments of the presented production models and their analysis are presented in Section 4. Finally, concluding remarks of the proposed study and possible research directions are provided in Section 5 of this study.

2. Literature review

We present a brief summary of current developments and trends in the literature stream of production-operations management in this section.

2.1. Serial production management system

Basic inventory models initiated by Harris [9] and Taft [10] are studied and modified extensively by the researchers and practitioners. Several studies performed on inventory management models consider single-stage production management. Literature analysis reveals that only a few papers with serial production system exist. Amongst these studies, Wein [11] provided a multi-stage manufacturing model considering random yield and reworking of defective items. General optimization techniques cannot always handle random yield problems because of their complex nature, and thus the development of heuristics to obtain optimal solutions become necessary. In this context, a heuristic for efficient solution of serial production system with random yield was provided by Grosfeld-Nir et al. [12]. Various reworking strategies were provided in Sarkar et al. [13]. Serial inventory model, and C´ardenas Barr´on [14] provided modified solution and extension of Sarkar et al. [13]. Impacts of learning and forgetting in a serial production management system were studied by Jaber and Khan [15] by considering that reworking process of defective items follows a learning curve. Their study was further extended by Glock and Jaber [16] by considering transferal of learning curve among successive batches. Tayyab and Sarkar [4] and Tayyab et al. [8] provided a serial inventory model incorporating random defective proportion and rework opportunity at each production stage of the production system. Seidgar et al. [17] suggested a maintenance-based scheduling for multi-stage assembly lines to improve reliability and production efficiency through a bi-objective algorithmic approach. Recently, Salmasnia and Shabanii [18] have proposed a novel maintenance system which focuses on bottlenecks in the multi-stage production system to reduce operational costs.

2.2. Imperfect quality production

In realistic manufacturing processes, both perfect-quality products and defective ones are produced. Thus, many researchers have modified conventional inventory models by considering constant defective rate, and others have considered stochastic defective proportion. Rosenblatt and Lee [19] investigated the manufacturing of perfect and imperfect products in an imperfect production system. Cheng [20] examined item production cost as its function of demand in an inefficient production system. Maity [21] analyzed time-dependent demand and variable production rate. Dey and Giri [22] investigated investment paybacks to decrease defective proportion while taking fluctuating demand rate into account. In an imperfect production process, Kumar and Goswami [23] concurrently investigated randomness and fuzziness. In an imperfect production process, Chen [24] discussed the replenishment cycle, number of shipments, and retailing pricing considerations of decaying items. An evolutionary algorithm-based strategy for a production management system with defects under after-sale warranty policies was presented by Taleizadeh et al. [25]. Further Khara et al. [26] have created a production model for an unreliable
production management that takes demand dependence on quality level and development cost into account. Dolai and Mondal [27] provided a production-inventory model to represent a system that starts with producing perfect products but shifts to imperfect production under a time-dependent manual inspection process and quality improvement cost. Recently, Elnaby et al. [28] combined machine learning with a quality process improvement method to significantly reduce defective production and material wastage in a plastic bottle production industry.

### 2.3. Random defective proportion

As with any production management system, the defective rate is not a fixed constant. This is why some academics have incorporated stochasticity of faulty proportion into their inventory models. Taking into account an inefficient production process that results in a randomly distributed number of damaged goods, Chiu et al. [29] proposed an approach for reducing the supplier’s inventory holding cost. By including a random defective proportion and process compressibility in their inventory model, Noorollahi et al. [30] developed a production plan with the highest likelihood of success. After factoring in process dependability as a deciding factor, Sarkar et al. [31] used control theory in a stochastic setting to provide a solution to the inventory model. In their multi-item inventory model, Chiu et al. [32] looked at the effects of a random defective production, scrap, rework policy, and multiple deliveries. Chiu et al. [6] used an algebraic technique to solve a multi-item inventory model with a random failing rate in order to find the appropriate common cycle duration, while Sarkar et al. [33] proposed a distribution-free method for enhancing quality and decreasing system setup costs. Decision support frameworks for imperfect production management systems with a random defective proportion have been created and their impacts on production decisions have been explored most recently by Sarkar et al. [34] and Tayyab et al. [8]. A research conducted by Pal and Adhikari [35] has examined the impact of random production defects on a multi-level supply chain and found that a longer buffer period before disruptions and shorter repair times benefit both the manufacturer and retailer in a coordinated system. Wang [36] provided an integrated optimization model for inventory and queuing management to improve equipment reliability resulting random defective proportion and increased production costs.

### 2.4. Defective rework opportunity

In a manufacturing system of imperfect nature, defective items are often fixed to convert them into acceptable quality products with some additional cost and effort. Many academics and industry experts have contributed valuable insights by detailing a wide range of redesign strategies for use in inventory simulations. Lee [37] used an inventory model that accounted for rework process setup durations, rework time variability, and corrective maintenance for machine breakdowns to estimate the ideal batch size. In their inventory model, Liu and Yang [38] accounted for both production loss and reworking opportunity. Chen and Tsao [39] looked at how lot size, learning effects, and the rework procedure affected a supply chain management model, and Inderfurth et al. [40] examined the degradation of defective items as they waited for the rework process. Depending on the severity of the initial failure, Pasandideh et al. [41] divided reworks into many categories. In a production environment where all faulty products are converted into acceptable quality products, Taleizadeh et al. [42] proposed multi-trade credit strategies. Chen [24] provided a two-tier supply chain management model with cooperation and non-cooperation rules to analyze the interaction between reworking and other process characteristics. Recent research by De et al. [43] and Shah and Naik [44] examined inventory models with incomplete production and rework cost information and unclear parametric information. Zhang et al. [45] compared a cell-based production system to a traditional assembly line system and found that the cell system is more reliable accounting of rework of defective products. Hasan et al. [46] analyzed the process through which retailers can reduce carbon emissions from their inventory. They examined the benefits of green technology investments and carbon pricing policies in managing inventory while considering factors including rework, demand, pricing, and promotions.

### 2.5. Price-dependent demand

The literature on inventory systems with variable demand rates often assumes that product demand varies in response to changes in supply or price, or sometimes both. Studies heading in this direction center on the central premise that lower prices result in increased levels of demand. It was assumed by Mondal et al. [47] that the product’s demand rate is linear in the seller’s asking price, and that its degradation rate is linear in the product’s time in storage. Mukhopadhyay et al. [48] conducted a similar analysis, focusing on the association between product demand and price and between the rate of degradation and storage duration. Ding [49] studied the impact of price-dependent demand in a production model, while Minner and Transchel [50] examined an EOQ model with price-dependent demand incorporating quantity discounts, providing optimum algorithmic solutions to their in-stock model. Both Sana [51] and Jaggi et al. [52] used
inventory models with trade credit financing that assumed that the demand data was an exponential function of the product price. In a two-tier production network, Kumar et al. [53] modeled the demand rate as a quadratic function of the product’s market price. To this end, they looked at both coordinated and non-coordinated supply networks. Ullah et al. [54] developed a dynamic pricing modeling approach for a price-sensitive demand inventory management system that takes into account the multi-period news-vendor situation. For a production-inventory model considering price-sensitive demand, Shaikh et al. [55] explored a two-tiered trade credit schema. Hanh and Chen [56] suggested a differentiation-based pricing strategy for price-sensitive new and remanufactured products that considers both consumer preference and their ease of substitution for each other. Adnan and Özelkan [57] studied how different pricing strategies can increase or decrease price fluctuations throughout a value chain. By studying different dependent-demand scenarios, they found that the way product demand responds to price changes and the type of uncertainty in that demand significantly impacts these fluctuations.

2.6. Monte Carlo simulation @RISK

New possibilities for expanding probabilistic techniques like Monte Carlo simulation in tackling complex issues have emerged as a result of the incredible increase in computer processing abilities and the development of pseudo-random number generators [58]. The term “Monte Carlo simulation” [59] refers to a group of methods for dealing with general stochastic issues by use of probability distributions and random numbers. Solving stochastic optimization issues with the application of Monte Carlo simulation has demonstrated effectiveness when combined with other analytical or heuristic optimization methodologies [60]. In contrast to deterministic issues, a major difficulty in the realm of the stochastic problems encountered is the unpredictable character of the variable, which takes on a new value with each inspection. Instead of computing the expectation of a random variable through a specific expectation formula, the Monte Carlo simulation method gets around this problem by using a set number of samples from which an estimate of the variable’s value at any given moment may be derived (which fixes the value of variable at one point in the given interval). In this context, Darmawan [61] developed a two-stage simulation-based method for designing supply chain networks that considers disruptions and improves system resilience by combining planning for normal operations with reactive strategies for disruption affected operations. Recently, Rashid and Mu’tasim [62] addressed a gap in hybrid flow shop scheduling by providing a simulation-based model that considers multiple cost factors in addition to the energy costs. For this, they introduced an improved algorithm that significantly reduces costs compared to existing methods by giving production planners a more powerful tool for cost-effective scheduling.

Monte Carlo simulation is widely employed in highly uncertain situations of parametric information for developing several (sometimes a million) distinct scenarios and feed the simulation outcomes into further model analysis. In realistic imperfect production situations, defective proportion continuously go on varying adopting unsteady values within some interval in all respects. Applying expectation formula from probability theory is not enough to handle the unsteady randomness in defective proportion of the imperfect production system. As per author’s knowledge, only a few production and supply chain models are available that have utilized this approach for handling the uncertainty while solving optimization models. Li et al. [63], Ma and Lv [64], and Rabbani et al. [65] are among those few studies who have taken advantage of this prosperous approach in production and supply chain modeling. Further, @RISK spreadsheet add-in for Microsoft Excel provides leading platform to utilize Monte Carlo simulation approach. Hence, our study takes advantage of @RISK 7.5 to handle uncertainty in random defective proportion and demand elasticity parameters for an imperfect serial production management system.

Rigorous literature survey indicates that a production decision framework for an imperfect serial production management with random defective rate and price dependent uncertain product demand considering a hybrid simulation-analytical optimization approach has not been reported yet. Hence, the proposed study considers an imperfect serial production management system to determine optimal batch quantity for the system under the impact of uncertain information regarding market response toward product price and uncertain shop floor conditions creating imperfections in the production management system by utilizing Monte Carlo simulation and analytical optimization technique @RISK. Figure 1 shows the contribution of proposed study in the production management literature and Figure 2 shows the relation among uncertainty, decision nodes, and the final objective payoff for the proposed production model through an influence diagram.

3. Mathematical model

This study examines the manufacturing of a single-product type inside an imperfect n-stage production management system, whereby all the stages operate as a serial production management system. While high-quality goods are manufactured at each level of
3.1. Assumptions

(1) An n-stage manufacturing process is considered that produces a single type of product.
(2) Production cycles are considered as relatively consistent [see [34]].
(3) Imperfect production proportion at each production stage is random in nature and follows beta probability distribution [see [4]].
(4) The time it takes to get prepared for production, known as setup time, is a fraction of the whole production time [66]. Each production stage’s setup time is considered as a p percent of the overall time for that stage.
(5) It is presumed that the costs and time spent transporting batches between later stages of production are insignificant.
(6) It is assumed that there is no capacity constraint on the production system’s storage capabilities [see [67]].

3.2. Model formulation

Taking into account the aforementioned nomenclature and assumptions, the following section presents three mathematical models considering annual dependent demand in response to changes in product price, as illustrated by the three types of relations given below.

(a) Linear relationship, \( \lambda = a - bp \), where parameter \( a \) and parameter \( b \) are considered as constants, \( a > 0 \) and \( b \geq 0 \).

This model shows a linear relationship between product price (\( p \)) and demand (\( \lambda \)), where the parameter (\( a \)) determines initial market size as considered by Tayyab et al. [4,8] and Sarkar et al. [34]. It represents the potential number of product units that can be sold if the price is set to zero (\( p = 0 \)). The parameter (\( b \)) describes the demand elasticity of the product as a non-negative constant (\( b \geq 0 \)) and depicts the sensitivity level of the product demand to the possible price changes. Here the term -bp represents a price effect, where product demand decreases in response to the increase in product price and vice versa. One can infer the below insights from this model.
(i) \( b = 0 \): No change in the product demand with any price fluctuations (perfectly inelastic product demand).

(ii) \( 0 < b < 1 \): Relatively inelastic product demand where changes in price induce a moderate effect on the demand.

(iii) \( b = 1 \): Proportional change in product demand with the change in price (unitary elasticity).

(iv) \( b > 1 \): Highly elastic product demand representing heavy changes in demand in response to the price changes.

This relationship between product price and demand assists managers in determining the optimal balance between profit maximization and expanding market reach [8]. Further, it incorporates the degree of price sensitivity of the products in informed decision making and thus the managers can apply dynamic pricing strategies such as offering discounts for highly sensitive products, whereas emphasizing on product differentiation and brand value for the low sensitive products at higher prices.

(b) Quadratic relationship, \( \lambda = a - bp - cp^2 \), where parameters \( a \), \( b \) and \( c \) are considered as constants, \( a \gg b > 0 \) and \( c > 0 \). This model is an extension to the linear price dependent demand and has been implemented by several studies including Kumar et al. [53] and Ullah et al. [54]. The terms \( bp \) and \( cp^2 \) in this model show how price changes affect demand. They’re influenced by baseline sensitivity (b) and sensitivity acceleration (c). Here, a higher value of b indicates that small increase in price will discourage many product buyers, and the parameter c indicates how the price effect gets stronger as the price increases. This model support managers in determining best price for their product. By considering the initial market size, how much sensitive customers are to the product price (b), and how quickly that demand sensitivity increases (c), decision makers can determine optimal trade off among profitability and market penetration [54]. In this way, the managers can avoid pricing their product too high or too low as it supports in determining the floor and ceiling prices of the product. The managers can visualize the change in demand with price adjustments as

(i) \( b \) much bigger than \( c \) (\( b \gg c \)): This indicates that small changes in price will be less likely impact on the product demand. This insight will be applicable for commodity products or those with few alternatives.

(ii) \( c \) bigger than \( b \) (\( c \gg b \)): This indicates that small changes in price can significantly impact the elastic demand. This is true for the case for luxury products with relatively higher number of substitutes.

c) Exponential relationship, \( \lambda = fp^e \), where \( f \) and \( e \) are positive constants.

This price-dependent demand model entails an exponential relationship among a product’s price (\( p \)) and demand (\( \lambda \)) as considered by Mondal et al. [47], Mukhopadhyay [48], and Sana [51]. This price-dependent demand model includes potential customer base (\( f \)) as an initial market size, the product’s price (\( p \)), and the exponential elasticity parameter (\( e \)) that reflects demand sensitivity to the changes in product price. The higher value of this elasticity parameter high elasticity (\( e > 1 \)) indicates drastic reductions in product demand in response to minute increase in the product price as seen in the case of luxury products. On the contrary, lower values of \( e \) (\( e < 1 \)) for nonsensitive products such as gasoline have minor demand swings as prices are fluctuated [48,51]. This insight enables managers to decide upon the best price setting mechanisms for profit maximization. The companies can determine a wise price point that strategically balances volume of demand and profit maximization through simultaneous consideration of product demand elasticity and cost of goods sold. Lowering the prices for highly elastic-demand products may result in a considerable increase in sales volume even if profit margins are reduced. However, price increases for inelastic or less-elastic-demand products may exhibit trivial effect on sales thus allowing for potentially higher profit margins.

Inventory behavior of \( k^{th} \) and \( n^{th} \) production stage is presented in Figures 3 and 4, respectively [see for instance [8]]. The percentage of defective products at each step of production is modeled as a random number, \( R \), with an expected value, \( E[R] \), in the design and analysis of production models. Here, we assume that \( R \) is a random number with a beta distribution [for instance, Tayyab et al. [8], with minimum and maximum values of 0 and 1, respectively. The restriction \( 0 < a < b < 1 \) must hold if both of these limits are to be taken into account as indicators of the fraction of defective products.
Expectation of the random variable \( R \) (following beta distribution) can be directly determined by implementing the basic explanation of expected value operator available in the theory of probability as

\[
E[R] = \frac{\alpha}{\alpha + \beta}
\]

Using the information presented in Figure 3, we can easily determine the whole inventory and the cycle time of production stage-\( k \). The accumulation of inventory at stage-\( k \) equals

\[
\mu_k = \frac{z^2(3\alpha_k\beta_k + \alpha_k^2 + \beta_k^2)}{2\omega_k(\alpha_k + \beta_k)^2},
\]

and total up-time of stage-\( k \) is

\[
E[TP(z)] = \frac{pz - \sum_{i=1}^{n} c_i \left( \frac{\alpha_{i_k}}{\alpha_{i_k} + \beta_{k_i}} + 1 \right) + z \sum_{i=1}^{n} j_i \left( \frac{\alpha_{i_k}}{\alpha_{i_k} + \beta_{k_i}} + 1 \right) + \sum_{i=1}^{n} K_i + \frac{1}{\lambda} z^2 \left( \sum_{k=1}^{n-1} \frac{2\alpha_k + \beta_k}{\omega_k(\alpha_k + \beta_k)} \right) + \frac{1}{\lambda} z^2 \left( \sum_{k=1}^{n-1} \frac{2\omega_k}{\omega_k(\alpha_k + \beta_k)} \right)}{2(p + 1)\omega_n(\alpha_n + \beta_n)^2 + \frac{1}{\lambda} z^2 \left( \sum_{k=1}^{n-1} \frac{2\omega_k}{\omega_k(\alpha_k + \beta_k)} \right)}.
\]

Under linear assumptions, the average inventory of the complete production system is comparable to the average inventory of finished goods in the system. In this way, we can calculate the overall production setup’s average inventory as

\[
\bar{\mu} = \frac{z(\alpha_n\beta_n(2\omega_n - 3\lambda) + \alpha_n^2(\omega_n - 3\lambda) + \beta_n^2(\omega_n - \lambda))}{2(p + 1)\omega_n(\alpha_n + \beta_n)^2(\lambda \sum_{k=1}^{n-1} \frac{2\omega_k}{\omega_k(\alpha_k + \beta_k)} + 1)}.
\]

Expected total profit of the system with constant demand rate (\( \lambda \)) including setup cost, production operations cost, reworking operations cost, product inspection cost, and storage cost is estimated as below.

\[
T_k = \frac{(p + 1)z(2\alpha_k + \beta_k)}{\omega_k(\alpha_k + \beta_k)}.
\]

Similarly, the entire inventory of stage-\( n \) can be determined using Figure 4.

\[
\mu_n = \frac{1}{2} z^2 \left( \frac{1}{\lambda} - \frac{3\alpha_n\beta_n + 3\alpha_n^2 + \beta_n^2}{\omega_n(\alpha_n + \beta_n)^2} \right),
\]

and total time including production, reworking, and inventory depleting time of the stage-\( n \) is

\[
T_n = \frac{(p + 1)z}{\lambda}.
\]

The total cycle time of the complete system can be estimated by the sum

\[
T = \sum_{k=1}^{n-1} T_k + T_n
\]

as below.

\[
E[TP(z)] = \frac{h\lambda z^2(3\alpha_n\beta_n + 3\alpha_n^2 + \beta_n^2) - \omega_n(\alpha_n + \beta_n)^2(hz^2 + 2\lambda(\delta - px + nx))}{2(p + 1)z(\lambda + 1)\omega_n(\alpha_n + \beta_n)^2}.
\]

**3.2.1. Case A**

The demand rate \( \lambda \) depends on the price with a linear relation \( a - bp \), where \( a > 0 \) and \( b \geq 0 \) [see for instance
In this case, modified expected total profit of the system is obtained as

\[
E[TP(z)]_{\text{linear}} = \frac{hz^2(a - bp)(3a_n\beta_n + 3\alpha_n^2 + \beta_n^2) - \omega_n(a_n + \beta_n)^2(2(a - bp)(\delta - px + \eta z) + hz^2)}{2(\rho + 1)z\omega_n(a_n + \beta_n)^2(ay - bpy + 1)}. \tag{9}
\]

Lemma 1. The profit function \(E[TP(z)]_{\text{linear}}\) has a maximum attainable value in the interval \([0, T]\).

Proof From the necessary conditions for optimum value of \(E[TP(z)]_{\text{linear}}\), first-degree differential of the profit function relative to the variable \(z\) is 0, as

\[
dE[TP(z)]_{\text{linear}} = \frac{hz^2(a - bp)(3a_n\beta_n + 3\alpha_n^2 + \beta_n^2) - \omega_n(a_n + \beta_n)^2(2a\delta + 2b\delta p + hz^2)}{2(\rho + 1)z^2\omega_n(a_n + \beta_n)^2(ay - bpy + 1)} = 0,
\]

which provides solution to the optimal batch size \(z^*_{\text{linear}}\) as

\[
z^*_{\text{linear}} = \left(\frac{2\delta(a - bp)}{p - \eta} - \sqrt{\frac{2\delta(a - bp)(a_n + \beta_n)^3}{\omega_n(a_n + \beta_n)^3} + (a - bp)^2}\right)
\]

To fulfil the sufficient conditions,

\[
\frac{d^2E[TP(z)]_{\text{linear}}}{dz^2} = -\frac{2\delta(a - bp)}{(\rho + 1)z^2(\eta(a - bp) + 1)}. \tag{12}
\]

It is obvious from the above that \(\frac{d^2E[TP(z)]_{\text{linear}}}{dz^2} < 0\), thus sufficient conditions for optimality of the profit function are satisfied. Hence, the profit function attends global maximum at \(z^*_{\text{linear}}\).

3.2.2. Case B

The demand rate \(\lambda\) depends on the price with a quadratic relation \(a - bp - cp^2\) where \(c > 0\), \(b > 0\) and \(a >> b\) [see for instance Mondal et al. (2003)]. In this case, modified expected total profit of the system is determined as

\[
E[TP(z)]^*_\text{linear} = \left[p - \eta - \frac{2\delta}{\sqrt{\frac{2\delta(a - bp)(a_n + \beta_n)^3}{\omega_n(a_n + \beta_n)^3} - (a - bp)(3a_n\beta_n + 3\alpha_n^2 + \beta_n^2)}}\right]
\]

Lemma 2. The profit function \(E[TP(z)]_{\text{quadratic}}\) has a maximum attainable value in the interval \([0, T]\).

Proof From the necessary conditions for optimum value of \(E[TP(z)]_{\text{quadratic}}\), first-degree differential of the profit function relative to the variable \(z\) is 0, as

\[
dE[TP(z)]_{\text{quadratic}} = \frac{hz^2(3a_n\beta_n + 3\alpha_n^2 + \beta_n^2)(a - p(b + cp)) - \omega_n(a_n + \beta_n)^2(2(a - p(b + cp))(\delta - px + \eta z) + hz^2)}{2(\rho + 1)z\omega_n(a_n + \beta_n)^2(ay - yp(b + cp) + 1)} = 0,
\]

\[
\frac{dE[TP(z)]_{\text{quadratic}}}{dz} = \frac{hz^2(3a_n\beta_n + 3\alpha_n^2 + \beta_n^2)(a - p(b + cp)) - \omega_n(a_n + \beta_n)^2(2\delta(p(b + cp) - a) + hz^2)}{2(\rho + 1)z^2\omega_n(a_n + \beta_n)^2(ay - yp(b + cp) + 1)} = 0, \tag{15}
\]
which provides solution to the optimal batch size \( z_{\text{quadratic}}^* \) as

\[
z_{\text{quadratic}}^* = \sqrt{\frac{2\delta(a - bp - cp^2)}{h(1 - \frac{(3a_0 + 3a_1^2 + \beta_0^2)(a - bp - cp^2)}{\omega_\eta(a + \beta_0)^2})}}.
\]

(16)

To fulfil the sufficient conditions,

\[
\frac{d^2E[TP(z)]_{\text{quadratic}}}{dz^2} = \frac{2\delta(p(b + c) - \eta)}{(\rho + 1)z^3(\eta y p(b + c) + 1)}.
\]

(17)

It is obvious from the above that \( \frac{d^2E[TP(z)]_{\text{quadratic}}}{dz^2} < 0 \), thus sufficient conditions for optimality of the profit function are satisfied. Hence, the profit function attains global maximum at \( z_{\text{quadratic}}^* \).

So, the maximum profit function of the system under the effect of quadratic dependence of demand on the product price is formulated as

\[
E[TP(z)]_{\text{quadratic}}^* = \left[p - \eta - \frac{2\delta}{h\omega_\eta(a + \beta_0)^2(a - bp - cp^2)} - \frac{2\delta h\omega_\eta(a + \beta_0)^2(a - bp - cp^2)}{(\rho + 1)z^2(\eta y p(b + c) + 1)}
\]

(18)

3.2.3. Case C

The demand rate \( \lambda \) depends on the price with an exponential relation \( fp^{-e} \) where \( f \) and \( e \) are positive constants [see for instance Jaggi et al. (2014)]. In this case, modified expected total profit of the system is determined as

\[
E[TP(z)]_{\text{exponential}}^* = \left[p - \eta - \frac{2\delta}{h\omega_\eta(a + \beta_0)^2(a - bp - cp^2)} - \frac{2\delta h\omega_\eta(a + \beta_0)^2(a - bp - cp^2)}{(\rho + 1)z^2(\eta y p(b + c) + 1)}
\]

(23)

Optimal batch size for each case discussed above is illustrated in Table 1, and the associated maximum profit functions are presented in Table 2.

4. Experimental case analysis

In this section, the change in optimal batch size and maximum profit relative to the proposed model parameters is evaluated using an experimental study based on numerical data presented as a case in Tayyab et al. [8].

4.1. Parametric information and solution methodology

Table 3 provides values of the model parameters with appropriate units for the experimental case analysis.
Table 1. Optimal batch size for different dependence types of demand on product price.

<table>
<thead>
<tr>
<th>Demand depends on product price</th>
<th>Batch size (items)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear relationship</td>
<td>( x_{\text{linear}} = \frac{\beta}{(\beta - \alpha)} \sqrt{\frac{2\alpha - \rho}{\beta - \alpha}} )</td>
</tr>
<tr>
<td>Quadratic relationship</td>
<td>( x_{\text{quadratic}} = \frac{\beta}{(\beta - \alpha)} \sqrt{\frac{2\alpha - \rho}{\beta - \alpha}} )</td>
</tr>
<tr>
<td>Exponential relationship</td>
<td>( x_{\text{exponential}} = \frac{\beta}{(\beta - \alpha)} \sqrt{\frac{2\alpha - \rho}{\beta - \alpha}} )</td>
</tr>
</tbody>
</table>

Table 2. Maximum profit for different dependence types of demand on product price.

<table>
<thead>
<tr>
<th>Relation</th>
<th>Maximum profit ( E[P(z)] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>( \frac{\beta}{(\beta - \alpha)} \sqrt{\frac{2\alpha - \rho}{\beta - \alpha}} \times \frac{(\beta - \alpha) - \rho}{(\beta - \alpha) + 1} )</td>
</tr>
<tr>
<td>Quadratic</td>
<td>( \frac{\beta}{(\beta - \alpha)} \sqrt{\frac{2\alpha - \rho}{\beta - \alpha}} \times \frac{e^{\frac{1}{\beta}} \times \beta^{\rho}}{(\beta - \alpha) + 1} )</td>
</tr>
<tr>
<td>Exponential</td>
<td>( \frac{\beta}{(\beta - \alpha)} \sqrt{\frac{2\alpha - \rho}{\beta - \alpha}} \times \frac{e^{\frac{1}{\beta}} \times \beta^{\rho}}{(\beta - \alpha) + 1} )</td>
</tr>
</tbody>
</table>

Table 3. Parametric information for the experimental case analysis.

<table>
<thead>
<tr>
<th>Parameter /production stage</th>
<th>First stage</th>
<th>Second stage</th>
<th>Third stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K ) ($/batch)</td>
<td>50</td>
<td>37</td>
<td>11</td>
</tr>
<tr>
<td>( c ) ($/item)</td>
<td>13</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>( j ) ($/item)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( \omega ) (items/year)</td>
<td>220,500</td>
<td>210,000</td>
<td>200,000</td>
</tr>
<tr>
<td>( a ) (triangularly distributed)</td>
<td>(0.02,0.03,0.04)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b ) (triangularly distributed)</td>
<td>(0.06,0.07,0.08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h ) ($/item/year)</td>
<td>5</td>
<td>45</td>
<td>5</td>
</tr>
<tr>
<td>( \rho ) ($/item)</td>
<td>45</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>( \omega ) (items/year)</td>
<td>220,500</td>
<td>210,000</td>
<td>200,000</td>
</tr>
<tr>
<td>( v ) (triangularly distributed)</td>
<td>(0.03,0.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f ) (triangularly distributed)</td>
<td>(0.015,0.03)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We have considered triangular distributions for the uncertain parameters \( (a, \beta, \alpha, b, c, e) \) as this distribution is flexible [68] and easy to use in the cases where exact information about the parameters that they are distributed is not available [69]. Triangular distribution provides efficient solutions for complicated systems that are inherently imprecise [70] as is the case of our proposed production management system where the defective proportion changes over time. Furthermore, triangular distribution can grasp the skewed data trends that are quite common in manufacturing systems [71]. Thus, this distribution supports in the cases where the complete and accurate parametric information is not available [68].

4.2. Solution methodology

Optimal batch quantity and maximum profit of the case production management system is obtained through below solution steps.

**Phase-1: Capturing the uncertainty in model parameters through Monte Carlo Simulation using @RISK spreadsheet modelling add-in of Microsoft Excel.**

**Step 1.1:** Compute expected values of the upper and lower bounds of random defective proportion \( E[a_{\text{min}, \text{m. likely}, \text{max}}] \) and \( E[b_{\text{min}, \text{m. likely}, \text{max}}] \) as RiskTrio\( \text{ng}[a, \text{\min}, \text{\m. likely}, \text{\max}] \) for each bound.

**Step 1.2:** Calculate expected value of defective proportion Access\( \text{is}, \text{deniedAccess\( \text{is}, \text{denied} \) at each production stage in @RISK from Step 1 and Monte Carlo simulation as = RiskBeta(\alpha, \beta).}
**Step 1.3:** Obtain expected values of the random demand elasticity parameters $E[b(min, m, likely, max)]$, $E[c(min, m, likely, max)]$, and $E[f(min, m, likely, max)]$ in @RISK as $\text{RiskTriang}(m,\text{likely},\text{max})$ for each elasticity parameter and compute expected product demand ($\lambda$) for each case.

**Phase 2: Determining optimal batch quantity ($z^*$) and profit of the system ($E[TP(z)]$)* through hybrid analytical technique and Monte Carlo Simulation approach.**

**Step 2.1:** Code optimal batch quantity ($z^*$) from Table 1 and optimal profit of the system ($E[TP(z)]$)* from Table 2 for each case in @RISK and designate these cells as risk output cells.

**Step 2.2:** Develop different scenarios of each case by running 100,000 iterations of the simulation.

**Step 2.3:** Record the mean values of ($z^*$) and ($E[TP(z)]$)* for each case after completion of the simulation.

**Step 2.4:** Assess the optimal conditions for maximum profit of the system ($E[TP(z)]$)* by analyzing sufficient conditions of optimality from the equations (10), (13), and (16) derived in the previous section, respectively.

### 4.3. Computational results

The proposed production model develops a hybrid analytical technique and Monte Carlo simulation-based approach to determine optimal batch size in a serial imperfect production management system under uncertain conditions. Experimental outcomes of the proposed model-cases are summarized in Table 4.

Optimal results for 100,000 scenarios presented in Table 4 indicate that the proposed serial production management system attains maximum mean profit ($\$519,576.80$ with a mean batch size of 1738.69 items) when the demand elasticity acquires linear relation with the product price (case A). Mean attainable profit in this case is $\$519,576.80$ with a 95th percentile value of $\$563,080.70$ and maximum profit level of $\$597,161.30$. Similar amount of profit is achievable in case B (where the product demand has a quadratic relation with the product price) when $E[c(min, m, likely, max)] \rightarrow 0$.

### 4.4. Discussion and analysis

It is obvious from the computational results of this study presented in Table 4 that the mean profit of case A is better than the other two cases. Variations among the obtained optimal solutions for profit maximization objective of the proposed model cases are obtained as

$$\text{Percentage gap} = \frac{\text{BMO} - \text{AMO}}{\text{AMO}} \times 100,$$

where AMO indicates Achieved Mean Output of the said case and BMO represents the Best Mean Output among all the model cases. This gap analysis indicates that the mean profit from case B and case C lag behind the mean profit from case A by 0.068% and 1.384%, respectively. Figure 5 illustrates the range of possible profit ($E[TP(z)]_{linear}$) from case A. Based on the above model outcomes, further analysis is performed on the results of case A with best achievable profit level.

#### 4.4.1. Effect of uncertain model parameters

The tornado diagram and spider graph of the sensitivity analysis of batch size for key random parameters of the model are represented in Figures 6 and 7, respectively. These illustrations indicate that the batch size is highly sensitive to the variation in lower bound ($\alpha$) of the random defective rate with a positive correlation coefficient of +0.91. Second important parameter affecting batch size is the upper bound ($\beta$) of random defective rate with a negative correlation coefficient of -0.38, whereas its correlation with demand elasticity parameter is -0.14.

One can observe that lower bound ($\alpha$) of the defective proportion is negatively correlated to the profit level of the system with a correlation coefficient of -0.92, whereas its upper bound ($\beta$) has a positive

| Table 4. Optimal batch quantity ($z^*$ items) and profit of the system ($E[TP(z)]$)* ($\$ per unit time)). |
|---------------------------------|-----------------|-------------|-----------------|-----------------|
| Attribute                      | Minimum         | Mean*       | Maximum         | 95th percentile |
| $z^*_{mean}$                   | 1690.89         | 1738.69*    | 1796.26         | 1710.96         | 1767.15         |
| $z^*_{quadratic}$              | 1689.28         | 1737.25*    | 1796.99         | 1709.55         | 1765.67         |
| $z^*_{exponential}$            | 1610.05         | 1710.33*    | 1830.11         | 1654.88         | 1768.21         |
| $E[TP(z)]_{mean}$              | 444,891.50      | 519,576.80* | 597,161.30      | 478,903.30      | 563,080.70      |
| $E[TP(z)]_{quadratic}$         | 444,615.60      | 519,225.70* | 596,739.00      | 478,582.70      | 562,695.40      |
| $E[TP(z)]_{exponential}$       | 430,552.00      | 512,484.60* | 606,098.40      | 470,665.30      | 556,875.30      |

Figure 5. The range of possible profit ($E[TP(z)]_{linear}$) from case A.
correlation coefficient of +0.38. This speaks for the focus of decision makers and managers towards making intensive efforts to reduce lower bound (α) of the defective production proportion in the entire production system which can be accomplished through lean tools implementation. Further, demand elasticity parameter (b) is negatively correlated to the optimal mean profit having a correlation coefficient of −0.02.

4.4.2. Sensitivity to the product price
In this section, the change in optimal profit of the system \(E[TP(z)]\) is analyzed by varying product price (p) from −50% to +25% in three equal intervals by utilizing RiskSimtable approach in the @RISK simulation modelling. Table 5 shows the change in profit of the system based on variation in the product price.

It is obvious from the conducted sensitivity analysis in Table 5 that 50% reduction in the given product price is not feasible for the analyzed serial production management system and 25% reduction in the given price reduces the mean optimal profit of the system by 66.16%. Further, 25% increment in the product price increases the mean optimal profit by 65.41%, but the product demand is reduced in this case, thus increasing the underutilized production capacity of the system. Therefore, the managers must set the product price vigilantly to align the corporate objectives with maximized capacity utilization.

4.4.3. Effect of variation in key cost parameters
This section analyzes sensitivity of optimal profit \(E[TP(z)]\) to the variation in key cost parameters of the proposed production management model. Values of the cost components including setup cost, order processing cost, and the inventory holding cost are individually varied from −50% to +50% and the corresponding changes in optimal profit of the system are studied in Table 6. This analysis indicates that mean profit of the proposed imperfect serial production management system is highly sensitive to the order processing cost \(c_i\), where 50% decrease in \(c_i\) increases the optimal...
Table 5. Sensitivity of the optimal profit to the product price.

<table>
<thead>
<tr>
<th>Change in price (p)</th>
<th>Minimum</th>
<th>Mean</th>
<th>Maximum</th>
<th>5th percentile</th>
<th>95th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>−50%</td>
<td>−228,970.50</td>
<td>−171,641.80</td>
<td>−111,930.30</td>
<td>−202,433.30</td>
<td>−138,823.30</td>
</tr>
<tr>
<td>−25%</td>
<td>109,528.23</td>
<td>175,844.00</td>
<td>245,540.70</td>
<td>140,213.20</td>
<td>213,942.90</td>
</tr>
<tr>
<td>+25%</td>
<td>772,973.60</td>
<td>859,411.10</td>
<td>940,211.30</td>
<td>814,170.40</td>
<td>907,641.60</td>
</tr>
</tbody>
</table>

Table 6. Sensitivity of $E[TP(z)]^{*}_{linear}$ to key cost parameters.

<table>
<thead>
<tr>
<th>Cost parameter</th>
<th>Percentage variation</th>
<th>New mean $E[TP(z)]^{*}_{linear}$</th>
<th>Percentage variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_i$</td>
<td>−50%</td>
<td>520,590.60</td>
<td>+0.20%</td>
</tr>
<tr>
<td></td>
<td>−25%</td>
<td>520,043.90</td>
<td>+0.09%</td>
</tr>
<tr>
<td></td>
<td>+25%</td>
<td>519,171.30</td>
<td>−0.08%</td>
</tr>
<tr>
<td></td>
<td>+50%</td>
<td>518,802.20</td>
<td>−0.15%</td>
</tr>
<tr>
<td>$c_i$</td>
<td>−50%</td>
<td>917,601.50</td>
<td>+76.61%</td>
</tr>
<tr>
<td></td>
<td>−25%</td>
<td>718,591.00</td>
<td>+38.30%</td>
</tr>
<tr>
<td></td>
<td>+25%</td>
<td>320,569.00</td>
<td>−38.30%</td>
</tr>
<tr>
<td></td>
<td>+50%</td>
<td>121,560.30</td>
<td>−76.60%</td>
</tr>
<tr>
<td>$h$</td>
<td>−50%</td>
<td>520,591.20</td>
<td>+0.20%</td>
</tr>
<tr>
<td></td>
<td>−25%</td>
<td>520,042.20</td>
<td>+0.09%</td>
</tr>
<tr>
<td></td>
<td>+25%</td>
<td>519,171.60</td>
<td>−0.08%</td>
</tr>
<tr>
<td></td>
<td>+50%</td>
<td>518,802.90</td>
<td>−0.15%</td>
</tr>
</tbody>
</table>

The mean profit by 76.61% and 50% increase in it reduces the optimal mean profit by 76.60%. On the other hand, the impact of $k_i$ and $h$ on mean profit of the system is trivial and similar to each other, where a 50% reduction in any of these parameters ($k_i$ and $h$) increases the mean profit by 0.20% and 50% increment in any of these reduces the mean profit by 0.15%. This change in optimal mean profit of the system suggests the managers and decision makers to primarily focus on reducing order processing cost in order to create more value from their respective imperfect serial production management systems.

4.4.4. Special case-I: model with a weighted average demand

A special case is created in this section with weighted average of elastic demand associated to aforementioned relations among demand and price of the product. In this case, optimal batch quantity ($z_{SC1}^{*}$) and profit of the system ($E[TP(z)]^{*}_{SC1}$) are computed as

\[
z_{SC1}^{*} = \frac{2\delta \times \langle w.\text{averagedemand} \rangle}{h \left( \frac{1}{(w.\text{averagedemand}) \times (3\alpha_i \beta_i + 3\alpha_i^2 + \beta_i^2)} \right)}.
\]

and

\[
E[TP(z)]^{*}_{SC1} = \left[ p - \eta - \frac{2\delta}{\langle w.\text{averagedemand} \rangle \times (3\alpha_i \beta_i + 3\alpha_i^2 + \beta_i^2)} \right] \times \left( \frac{\langle w.\text{averagedemand} \rangle}{\langle w.\text{averagedemand} \rangle + 1} \right).
\]

Table 7 shows the mean simulation results of the special case with 100,000 different scenarios. In this case, mean optimal batch quantity of 1728.71 items generate a mean profit of $517,124.30 which is lagging behind the optimal profit from case A by 0.47% only. The benefit of adopting the batch quantity of this case is worth the uncertainty in model parameters, as it is capturing all the three possible types of uncertain demand dependencies on the product price with a trivial cutback in optimal profit of the system. Therefore, the managers can get advantage of profit maximization through implementation of the proposed decision framework in an imperfect serial production management system under highly uncertain environment.

4.4.5. Special case-II: model with variable transportation costs between multi-stages

In this section, we relax the Assumption 5 by considering a variable batch transportation cost ($v_i, \forall i = 1, 2, 3, \ldots, n-1$) among the production stages. Hence, the profit maximization function for Case-I is modified as below

\[
E[TP(z)]^{*}_{linear} = \sum_{i=1}^n v_i(a - bp) + \frac{h z^2 (a - bp)(3\alpha_n \beta_n + 3\alpha_n^2 + \beta_n^2) - \omega_n (\alpha_n + \beta_n)^2 (2(a - bp)(\delta - p\eta) + h z^2)}{2(p + 1)\omega_n (\alpha_n + \beta_n)^2 (ay - bpy + 1)}.
\]

Table 7. Optimal batch quantity ($z^{*}$ (items)) and profit of the system ($E[TP(z)]^{*}$ ($ per unit time)) for Special Case-1 of the proposed production model.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Minimum</th>
<th>Mean</th>
<th>Maximum</th>
<th>5th percentile</th>
<th>95th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{SC1}^{*}$</td>
<td>1665.86</td>
<td>1728.71</td>
<td>1801.99</td>
<td>1697.20</td>
<td>1761.12</td>
</tr>
<tr>
<td>$E[TP(z)]^{*}_{SC1}$</td>
<td>440,508.20</td>
<td>517,124.30</td>
<td>598,520.10</td>
<td>476,647.50</td>
<td>560,579.30</td>
</tr>
</tbody>
</table>
After applying analytical optimization approach by satisfying necessary and sufficient conditions of optimality, we derive optimal batch size \( z_{SC}^* \) and maximum profit of the system as below.

\[
z_{SC}^* = \left[ \frac{2\delta(a - bp)}{h(1 - (a - bp)(a_0 + \beta_0)^2)} \right].
\]

and

\[
E[TP(z)]^*_{SC} = \left[ p - \eta - \frac{2\delta}{h_{w_0}(a_0 + \beta_0)^2 - h(a - bp)(3a_0\beta_0 + 3a_0^2 - \beta_0^2)} \times \frac{a - bp}{(p + 1)(y(a - bp) + 1)} - \sum_{i=1}^{n} v_i(a - bp) \right].
\]

Table 8 summarizes the optimal policy by considering batch transportation cost among different production stages. One can observe that the inter-stage transportation cost has not affected the optimal batch size, but it has significantly reduced the mean profit by $17068.0, which is a 3.28% reduction in profitability of the system. Managers can utilize this special case results to identify the impact of current manual or semi-automated batch transfer systems among the production stages and use the insights from this model in evaluating the feasibility of automated batch transfer equipment through present value analysis.

### 4.4.6. Special case-iii: model with storage capacity constraints

In this section, we relax Assumption 6 of the proposed model for Case A by considering capacity constraints at each production stage and provide a heuristic to determine the optimal policy where each production stage has a maximum storage capacity of \( S_i \) (\( i = 1, 2, 3, \ldots, n \)) units. The solution heuristic for this special case is as follows.

\[
\text{for } i \in (1, 2, 3, \ldots, n), \text{ calculate } z_{linear}^* \text{ from Equation 9 as } z_{linear}^*(1), \text{ record } z_{linear}^*(1) \text{ and calculate } E[TP(z)]^*_{linear} \text{ from } \text{Equation 11 as } E[TP(z)]^*_{linear}(1), \text{ record } E[TP(z)]^*_{linear}(1) \text{ if } z_{linear}^*(1) \leq S_i \forall i, i = (1, 2, 3, \ldots, n)
\]

\[
\text{return } z_{linear}^* = z_{SC3}^*, \text{ and } E[TP(z)]^*_{linear} = E[TP(z)]^*_{SC3}
\]

else

\[
\text{return } min(S_i) = z_{SC3}^*, \text{ and } E[TP(z)]^*_{CaseA} = E[TP(z)]^*_{SC3}
\]

end

The aforementioned heuristic presents a dual support for managers and decision makers in adjusting the batch size as per bottleneck production stage capacity when \( z \leq S_i \forall i, i = (1, 2, 3, \ldots, n) \), and in designing the minimum storage space requirements for each production stage of the system when \( z \leq S_i \forall i, i = (1, 2, 3, \ldots, n) \).

### 4.5. Managerial insights

The proposed production management model considers both market and shop-floor uncertainties while determining the optimal production policy for an imperfect serial production management system. Managers of such production management systems can directly benefit from the closed form solutions obtained through this model to capture the uncertain parameters and maximize total profit. Model analysis indicates that the optimal profit is substantially sensitive to the lower bound of defective proportion and the order processing cost. Hence, managers should pay close attention toward essentially reducing the order processing cost first to improve the profit that can be achieved through various means but not limited to the productivity improvement techniques, employee training, and acquiring efficient ways to cut down product conversion costs.

An unusual and interesting insight worth consideration is the similar impact of setup cost and finished items holding cost on the optimal profit of the system. This aspect of the presented production inventory model enables the managers to become indifferent in their decision to pick any of these costs first to work on with an objective of improving value of the system. In addition, development of 100,000 different experimental scenarios of uncertain conditions through Monte Carlo simulation using @RISK simulation modeling in the proposed model supports the managers to capture almost each and every possible realization of the real world production scenarios and strengthen their batch size decisions. Further, the special case formulation considers equal weighted average

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Minimum</th>
<th>Mean</th>
<th>Maximum</th>
<th>5th percentile</th>
<th>95th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_{SC} )</td>
<td>1690.89</td>
<td>1738.69*</td>
<td>1798.26</td>
<td>1710.96</td>
<td>1767.15</td>
</tr>
<tr>
<td>( E[TP(z)]^*_{SC} )</td>
<td>427,823.50</td>
<td>502,508.80*</td>
<td>580,093.30</td>
<td>461,835.30</td>
<td>546,012.70</td>
</tr>
</tbody>
</table>
scenario of different market conditions. The managers are recommended to select this solution for application as this special case provides a single solution for all the three types of possible demand dependencies on the product price with a minor drop (≈0.47%) in the mean system profit.

5. Concluding remarks

Decision making in a real-world serial production scenario is a challenging task, where managers have to consider several types of internal as well as external uncertainties. As the product type and production process itself become complex, these uncertainties evolve as extensive barriers in determining favorable production policy to maximize value of the system. Random defective proportion acts as a prevailing internal uncertainty [54] and price-dependent random demand is a dominant external uncertainty [55] to deal with. In this direction, Tayyab et al. [8] have provided a production decision model for an imperfect serial production system by incorporating a real workplace condition of random defective proportion at each production stage. Their production model has adequately grasped the internal uncertainty of the system, but they over passed the external uncertainty in product demand. This paper tends to bridge this literature gap by extending their model considering price-dependent product demand.

The proposed model provides three different pragmatic cases considering varied uncertain relationships among product demand and price, and thus, three distinct mathematical models are formulated. Closed form solution of the proposed cases is obtained through analytical optimization technique, and @RISK simulation modeling approach is utilized to capture the uncertainty in the random imperfect proportion and demand information by developing extensive 100,000 scenarios through Monte Carlo simulation. Experimental outcomes of the case analysis verify the practicality of the proposed model through sensitivity analysis for various real-world instances and several useful insights are obtained. Managers of the serial production management systems can take advantage of the proposed decision framework by its direct implementation as it considers significant internal and external uncertain scenarios of the system and attains optimal outcomes through a hybrid analytical technique and @RISK simulation approach.

Besides, the proposed decision model has some limitations in its applicability, which include non-consideration of the backorders in the situations of variations in lead time. Thus, a simple extension to this model can be the consideration of backorders. In order to illustrate more realistic shop-floor conditions, some other significant considerations can be incorporated in this production model. Production inspection is essential for every production system, thus, design of inspection schema can be incorporated in this inventory model. Interruptions in the production system and service level constraint for quality improvement and setup cost reduction can serve as some of the advanced future extensions to this research work.

Nomenclature

<table>
<thead>
<tr>
<th>Decision variable</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>batch size (in items)</td>
</tr>
<tr>
<td>$n$</td>
<td>total production stages (integer)</td>
</tr>
<tr>
<td>$p$</td>
<td>product price ($/item)</td>
</tr>
<tr>
<td>$\omega_k$</td>
<td>stage-$k$ production rate (items/time)</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>stage-$n$ production rate (items/time) ($\omega_k &gt; \omega_n &gt; \lambda$)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>demand rate (items, price-dependent)</td>
</tr>
<tr>
<td>$E[R_i]$</td>
<td>expectation of defective production at stage-$i$ (items)</td>
</tr>
<tr>
<td>$T_k$</td>
<td>stage-$k$ cycle length (year)</td>
</tr>
<tr>
<td>$T_n$</td>
<td>stage-$n$ cycle length (year)</td>
</tr>
<tr>
<td>$T$</td>
<td>total cycle length of the system (year)</td>
</tr>
<tr>
<td>$k_i$</td>
<td>stage-$i$ setup cost ($/setup$)</td>
</tr>
<tr>
<td>$c_i$</td>
<td>stage-$i$ production cost ($/item$)</td>
</tr>
<tr>
<td>$j_i$</td>
<td>stage-$i$ inspection cost ($/item$)</td>
</tr>
<tr>
<td>$v_i$</td>
<td>variable batch transportation cost from stage $i$ to stage $i + 1$ ($/item$)</td>
</tr>
<tr>
<td>$S_i$</td>
<td>storage capacity of stage $i$</td>
</tr>
<tr>
<td>$h$</td>
<td>cycle inventory holding cost ($/item/\text{cycle}$)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>setup time proportion of each stage (percentage)</td>
</tr>
<tr>
<td>$\hat{\mu}$</td>
<td>cycle inventory average (items)</td>
</tr>
<tr>
<td>$E[TP(z)]$</td>
<td>expected total profit of the entire system ($/time$)</td>
</tr>
</tbody>
</table>

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Notes on contributors

Muhammad Tayyab is an Assistant Professor in the Information Systems and Operations Management Department and the Interdisciplinary Research Center for Finance and Digital Economy at King Fahd University of Petroleum and Minerals (KFUPM) in Dhahran, Saudi Arabia. His research interests include supply chain management, operations management, and big data analytics.

Asif Iqbal Malik is an Assistant Professor in the Department of Industrial and Systems Engineering and the Interdisciplinary Research Center for Smart Mobility and Logistics at KFUPM. His research interests include operations management, logistics, and supply chain management.
Irfanullah Khan is a post-doctoral fellow at the Interdisciplinary Research Center for Smart Mobility and Logistics at KFUPM. His research interests include operations management, logistics, and risk analysis.

Mehran Ullah is a lecturer at the School of Business and Creative Industries at the University of the West of Scotland in Paisley, Scotland. His research interests include supply chain management, circular economy, and digital processes.

Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

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