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A group decision-making algorithm considering interaction and feedback mechanisms for dynamic supplier selection under q-rung orthopair fuzzy information

Abstract:
Supplier selection is vital for enterprises to operate stably and achieve and sustain a competitive advantage. However, from the initial establishment to the gradual development and maturity of the enterprise, the supplier selection criteria change dynamically, and decision-makers are hardly in agreement in each stage, which creates challenges for enterprises when choosing suitable suppliers. As such, this paper proposes a multicriteria group decision-making method based on an interaction and feedback mechanism (IFM) and q-rung orthopair fuzzy sets (q-ROFS) theory. We introduce an interaction and feedback mechanism to achieve consensus among different decision-makers in group decision-making. We then develop the q-rung orthopair fuzzy weighted partitioned Bonferroni mean (q-ROFWPBM) operator to address the aggregation problem of dynamic multicriteria. A group decision-making algorithm combining the IFM and q-ROFWPBM operator is proposed to analyze the Hongxing Erke supplier selection problem. The results show that the proposed method can not only account for large differences of opinion among decision-makers during group decision-making but also consider the dynamic changes of supplier selection criteria in different stages of enterprise development and help enterprises choose suppliers suitable for their own development characteristics.

Key words: dynamic supplier selection; interaction and feedback mechanism; q-rung orthopair fuzzy set; multicriteria group decision-making

1. Introduction
A supply chain refers to a functional network chain structure around the core enterprise, starting from supporting parts, making intermediate products and final products, and finally sending products to consumers by the sales network, including suppliers, manufacturers, distributors, and end users. Suppliers are the source of the supply chain, and their selection process is an important link in the process of supply chain management. It has a direct and vital impact on the performance and sustainable development of an enterprise. In today’s market, the supply chain of apparel and other enterprises has the characteristics of unpredictable and changeable customer demand, short product life cycle, wide variety, and long supply process. Therefore, to improve their product competitiveness and operation performance in a complex and changeable market, clothing enterprises must correctly select the most appropriate suppliers. The performance of suppliers directly affects the quality of products, which then affects the comprehensive performance of enterprises.

When an enterprise is at different stages of development, the supplier requirements are also different. For example, in the early stage of development, if the company does not have enough capital, suppliers must have a lower cost while ensuring product quality; in the company’s development period, the enterprise has a certain capital and has lower requirements for the cost of suppliers but may have higher requirements for supply efficiency; in the mature period of enterprise development, in addition to requiring suppliers to have higher supply efficiency, they also need to
have sufficient resilience to respond to sudden changes in the market environment. Therefore, the enterprise’s supplier selection criteria are constantly changing, and the supplier selection process is a dynamic selection process. Founded in 2000, the Hongxing Erke group (https://www.erke.com/) is a large sports and leisure clothing enterprise integrating R&D, production, and sales in China. In July 2021, Henan Province in Middle China suffered a catastrophic flood, and many well-known enterprises donated to the disaster area. Hongxing Erke quietly donated 50 million materials under the condition of long-term performance losses. This incident immediately brought attention to Hongxing Erke. Many consumers bought Hongxing Erke products out of praise and admiration for this behavior. The next day, sales alone increased by 52 times year-on-year. Due to the sudden demand, Hongxing Erke stores and online stores were in short supply, stores on Taobao.com also lacked human and material resources, and delivery was delayed for approximately a week. Because the supplier did not have sufficient supply, it was difficult to cope with the sudden surge of orders, which also caused great pressure on Hongxing Erke’s supply chain. In addition, it adversely affected the development of enterprises. If Hongxing Erke’s suppliers have good adjustment ability and adaptability, they can alleviate the shortage problem in time and ensure the efficiency of the product supply. Otherwise, the supply chain may enter a state of stagnation or paralysis and hinder the development of the enterprise. The selection of suppliers needs to comprehensively consider all aspects of capabilities and establish evaluation indicators, which has always been a hot issue for scholars.3,4,5

Overall, the evaluation criteria for supplier selection must consider the actual situation and consider the requirements of enterprises for suppliers at different development stages. To handle the above problems and select the most suitable supplier for similar enterprises represented by Hongxing Erke, we propose a group decision-making method based on the interaction and feedback mechanism (IFM) and q-rung orthopair fuzzy set, study its application in specific cases, and verify the effectiveness of this method through sensitivity analysis and comparative analysis. The main contributions of this paper are as follows:

- The interaction and feedback mechanism is established to solve the consensus problem among group decision-makers in the group decision-making problem.
- We combine the idea of partitioning with fuzzy set theory to develop the q-rung orthopair fuzzy weighted partitioned Bonferroni mean (q-ROFWPBM) operator to solve the dynamic change problem of supplier selection criteria in the process of enterprise development.
- A group decision-making framework based on the IFM and q-ROFWPBM operator is established to provide decision support for dynamic supplier evaluation and selection.

The rest of this paper is as follows: Section 2 reviews and summarizes the literature on supplier selection methods and fuzzy group decision-making; Section 3 puts forward the IFM, the q-ROFWPBM operator, and their related basic knowledge; in Section 4, a multicriteria group decision-making framework based on the IFM and q-ROFWPBM operator is established; in Section 5, the method proposed above is applied to the dynamic supplier selection of Hongxing Erke, and the effectiveness of the method is verified by sensitivity analysis and comparative analysis; and Section 6 summarizes the conclusion.

2. Literature review

2.1. Supplier selection

In a complete supply chain, the choice of suppliers as the source is very important for enterprises.
Choosing a reliable supplier will not only reduce various problems in the supply chain but also promote the development of enterprises. Many scholars have studied the selection of enterprise suppliers and developed various selection methods. A large part of these methods is to select the evaluation criteria first and then select the supplier through multicriteria decision-making. Many scholars have applied fuzzy theory to multicriteria decision-making to study the problem of supplier selection. Kusi-Sarpong et al. proposed a two-stage supplier selection decision model based on fuzzy set theory, where the model adopts a scientific method and incorporates performance criteria when screening and selecting potential suppliers to further optimize supplier selection. A case study of a beverage production company in Ghana, sub Saharan Africa proves the effectiveness of this method. Chen et al. proposed a hybrid rough fuzzy decision making trial and evaluation laboratory method, which combines the advantages of a fuzzy set in dealing with internal uncertainty and a rough set in dealing with external uncertainty. The effectiveness and accuracy of this method are illustrated by its application in sustainable vehicle transmission supplier selection and comparison with other methods. Liu et al. extended Schweizer Sklar t-conorm and t-norm to intuitionistic fuzzy numbers, proposed two multicriteria group decision-making methods, and applied them to a multiattribute group decision-making problem for supplier selection. Pang et al. proposed a method combining fuzzy set theory and gray correlation analysis. The weight of each criterion is calculated by gray correlation analysis, the fuzzy comprehensive evaluation method is improved, and the suppliers are compared by using the membership function of normal distribution. Rabbani et al. proposed a new interval valued fuzzy group decision model to evaluate the sustainability performance of suppliers in sustainable supply chain management, which can help supply chain experts or decision-makers systematically determine the preference order of sustainable supplier candidates under uncertain conditions. Tooranloo et al. proposed an AHP method combined with interval intuitionistic fuzzy sets to select appropriate suppliers in a group decision-making environment. Wang et al. proposed a green supplier selection method based on a q-ROF fuzzy set, which combines the q-ROFPWA operator with the todim method based on prospect theory. An example of supplier selection of an electric vehicle company shows the effectiveness of this method. Liu et al. proposed a multicriteria group decision-making method based on a generalized ordered weighted hesitation fuzzy priority weighted average operator for green supplier selection. In addition to applying fuzzy theory, many scholars have developed some other multicriteria supplier selection methods. Guarnieri et al. first used the Copeland method to extract evaluation criteria, then used the AHP method to obtain decision-makers’ views on criteria and weight definitions of these criteria, and finally used the electre-tri method to classify suppliers according to multiple criteria. Kaya et al. proposed a multicriteria decision-making method integrating Bayesian network, called decision test and evaluation laboratory, and applied it to the supplier selection of a large automobile manufacturer in Turkey. The method proved suitable for the decision-making problems with multicriteria, high uncertainty, and limited data. Lo et al. integrated the best-worst method, an order preference correction fuzzy technology based on ideal solution similarity (TOPSIS) and fuzzy multiobjective linear programming to solve the problems of green supplier selection and order allocation. Amiri et al. proposed a new supply chain sustainable supplier selection model based on the triangular fuzzy method, and proved the effectiveness of the method with a real automobile supplier selection case. Song et al. proposed an integrated method based on the pairwise comparison method, DEMATEL, and rough set theory, then took a solar air conditioner manufacturer as an example to verify the effectiveness. Liou et al. developed a data-driven MADM method.
model that combined a random forest algorithm, decision experiment and evaluation laboratory, and multiojective optimization based on expected level ratio analysis. The model utilized potential rules/patterns derived from a large amount of historical data to help decision-makers objectively select suitable green suppliers. There have been many studies on supplier selection, and the proposed methods are also diverse. These methods were applied to supplier selection decisions in different industries. The facts have proven that a good decision-making method can help companies choose the most appropriate suppliers and promote the long-term development of the enterprise.

2.2 Fuzzy group decision-making

Usually, many problems in real life do not have an accurate definition criterion and have uncertainty and fuzziness. The same is true of the decision-making process. There needs to be a specific way to represent the fuzzy evaluation value in the decision-making process. Zadeh proposed a fuzzy set, which expressed the degree of certainty of a thing in the form of probability, that is, membership \(u\). However, the fuzzy set only defined membership \(u\) and ignored rejection. Therefore, Atanassov proposed a concept of an intuitionistic fuzzy set, which included membership \(u\) and nonmembership \(v\) to represent satisfaction and rejection in the decision-making process. After that, intuitionistic fuzzy sets were widely used in different decision problems. Because the application condition of the intuitionistic fuzzy set is \(0 \leq u + v \leq 1\) and the situation of \(u + v > 1\) is not considered, Yager proposed the Pythagorean fuzzy set, which satisfied \(u + v > 1\) and \(0 \leq u^2 + v^2 \leq 1\), which solved this problem and has a wider application range. Later, Yager proposed the q-rung orthopair fuzzy set, which satisfied \(0 \leq u^q + v^q \leq 1\) and \(q \geq 1\), which further expanded the spatial representation range of information. Later, many scholars extended and further studied the q-rung orthopair fuzzy set and developed different multiattribute decision-making methods, and the application scope was expanded. Zhang et al. studied a method to determine priority weight from individual or group q-rung orthopair fuzzy preference relations (q-rofprs). Peng et al. proposed a q-rung orthopair fuzzy decision method based on weighted distance approximation (wdba), in which the weight of the decision-maker was obtained from the nonlinear optimization model according to the deviation-based method, and proposed a new q-rung orthopair fuzzy number (q-ROFN) score function to solve the failure problem when comparing two q-ROFNs. Liu et al. proposed the distance measure between q-rung orthopair hesitation fuzzy sets based on the concept of “multiple fuzzy sets”, and developed the TOPSIS method for the proposed distance measure. Hussain et al. proposed a method to deal with mixed q-rung orthopair fuzzy sets covering rough sets and TOPSIS concepts, which has a stronger ability to manage uncertainty. The MCDM methods combining q-ROFs with information aggregation operators are also very rich, such as q-ROF Muirhead mean operators, q-ROF Hamy mean operators, and q-ROF Heronian mean operators. In addition, Bonferroni proposed the Bonferroni mean (BM) operator, which was used to express the mutual influence relationship between any two evaluation criteria, and it has been widely used in the field of multicriteria decision-making. Wang et al. combined the BM operator with a 2-Tuple linguistic neutrosophic number to explore its application in green supplier selection. Liu and Teng explored the application of the BM operator in the interval neutrosophic hesitant uncertain linguistic environment. Liu et al. combined a multivalued neutrosophic number with the BM operator to explore the application. Wei et al. combined the generalized weighted Bonferroni geometric mean operator with interval neutrosophic numbers to develop a multicriteria model.
decision algorithm. However, in some cases, not all objects have mutual influence relations, but partition objects have mutual relations and need to be divided into different parts. Therefore, some scholars have proposed partitioned operators and explored their applications. Liu et al. proposed an intuitionistic fuzzy partitioned BM operator. Liu et al. proposed the partitioned Heronian mean operator of linguistic intuitionistic fuzzy numbers. The proposal and application of these methods enriched the existing multiattribute decision-making theory and provided a solution to the multiattribute decision-making problem within a complex environment.

The combination of fuzzy sets and aggregation operators is also widely used in multiattribute group decision-making. Liu et al. proposed a weighted partitioned Maclaurin symmetric average (IFWPMSM) operator based on intuitionistic fuzzy numbers to solve the problem of multiattribute group decision-making. Garg and Chen proposed a weighted average neutral aggregation operator (AOS) and combined it with q-ROFs to establish a multiattribute group decision-making method. Mu et al. proposed a weighted interval Pythagorean fuzzy Mclauglin mean operator (IVPFMSM) to solve the complex multiattribute group decision-making problem (MAGDM) involving the interaction between unreasonable evaluation values and input parameters. Liu et al. proposed a new concept called a complex q-rung language set to deal with complex uncertain information in practical decision-making problems, combined it with the HM operator, proposed a variety of operators, and proved its application in multiattribute group decision-making. Liu and Liu proposed a multiattribute group decision-making method based on q-ROFs and the BM operator. Li et al. combined intuitionistic fuzzy sets with a hammy mean (HM) operator to propose a multiattribute group decision-making method. Zhang explored the application of aggregation operators based on trapezoidal interval type-2 fuzzy sets in multiattribute group decision-making problems. Hendiani et al. combined interval type-2 trapezoidal fuzzy sets and BM operators to construct a multiattribute group decision-making method and used consistent lower and upper likelihoods to improve the efficiency of the group decision-making framework. Chen et al. presented a new method for fuzzy multiple attributes group decision-making based on the proposed ranking method of trapezoidal interval type-2 fuzzy sets and the TOPSIS method. In addition, in the process of multiattribute group decision-making, the consensus of decision-makers is very important. Many scholars have studied the consensus reaching process in large-scale group decision-making. Xu et al. proposed a large-scale group decision-making model based on trust consensus by first combining the rationality and uncertainty of adjustment information to construct a confidence level to measure the fairness and objectivity of adjustment information, and then by establishing a mechanism to solve noncooperative behavior. Urena et al. mixed the process of expressing closer opinions in the consensus building process with the evolving relationship between experts based on social network analysis, which solved the problem that the consensus building process alone may not result in agreement. Xiao et al. and Xu et al. proposed two-stage consensus reaching models to establish consensus among different clusters. Chao et al. introduced cosine similarity to construct the distance measure of different preference structures, and used cluster analysis to divide large-scale groups and deal with noncooperative behavior, which reduced the impact of noncooperative behavior and promoted consensus achievement. Gou et al. developed a large scale group decision-making consensus reaching process with a two-level hesitation fuzzy language preference relationship, and proposed a clustering method based on similarity, a weight determination method based on two-level information entropy, and a consistency measurement method to ensure the consensus reaching process. Tang et al. studied the consensus reaching
process in a heterogeneous large-scale group decision-making environment. Combined with the k-means algorithm, they proposed an ordered consistency measure with an objective threshold based on preference ranking and gave an example to verify the effectiveness of the model. Wu and Xu combined the hesitant fuzzy elements based on possibility distribution with a k-means clustering method to propose a large-scale group decision consensus model. With the change of cluster (virtual or nominal) in each interactive consensus round, the evolution of the consensus process can be captured.

In summary, the existing research on supplier selection methods is very rich, but few studies consider the different needs of enterprises in different stages of development. Therefore, this paper combines q-ROFs with a partitioned BM operator and combines the interaction and feedback mechanism to propose a multicriteria group decision-making method, which not only solves the consensus problem in the process of group decision-making but also accounts for the dynamic changes of supplier requirements during enterprise development and provides a theoretical method for enterprises to select an appropriate supplier.

3. Preliminaries

In this section, the q-rung orthopair fuzzy set, the interaction and feedback mechanism of group decision-making in the environment of the q-rung orthopair fuzzy set are introduced, the q-rung orthopair fuzzy partitioned Bonferroni mean (q-ROFPBM) operator and q-ROFWPBM operator are proposed, and their algorithms and properties are explored.

3.1 The q-rung orthopair fuzzy set

Definition 1. (Yager) Let \( X \) be a fixed set; a q-ROF \( A \) on \( X \) can be represented as

\[
A = \{ (x, \mu_A(x), \nu_A(x)) | x \in X \}
\]

where \( \mu_A(x) \in [0,1] \) denotes the degree of membership and \( \nu_A(x) \in [0,1] \) denotes the degree of nonmembership of the element \( x \in X \) to the set \( A \), respectively, with the condition that

\[
0 \leq (\mu_A(x) + \nu_A(x)) \leq 1, (q \geq 1) \quad .
\]

The degree of indeterminacy is given as

\[
\pi_A(x) = (1 - \mu_A(x) - \nu_A(x))^{\frac{1}{q}}. 
\]

For convenience, we called \((\mu_A(x), \nu_A(x))\) a q-ROF denoted by \( A = (\mu_A, \nu_A) \).

Definition 2. (Liu et al.) Let \( a_1 = (u_1, v_1) \) and \( a_2 = (u_2, v_2) \) be two q-ROFs and \( \lambda \) be a nonnegative real number; then,

1. \( a_1 + a_2 = (u_1 + u_2 - u_1 u_2, v_1 + v_2) \)

2. \( a_1 \times a_2 = (u_1 u_2, v_1 + v_2 - v_1 v_2) \)

3. \( \lambda a_1 = (1-(1-u_1)^{\frac{1}{q}}, v_1^{\frac{1}{\lambda}}) \)
\[
(4) \ a_i^b = \left[ u_i^a \left( 1 - \left( 1 - v_i^a \right)^\frac{1}{\lambda} \right)^\frac{1}{\lambda} \right]
\]

**Definition 3.** (Liu et al.) Let \( a = (u_a, v_a) \) be a q-ROFN; then, the score function of \( a \) is defined as
\[
S(a) = u_a^a - v_a^a,
\]
and the accuracy function of \( a \) is defined as
\[
H(a) = u_a^a + v_a^a.
\]
For any two q-ROFNs \( a_1 = (u_1, v_1) \) and \( a_2 = (u_2, v_2) \), then,

1. If \( S(a_1) > S(a_2) \), then \( a_1 > a_2 \);
2. If \( S(a_1) = S(a_2) \), then
   
   - If \( H(a_1) > H(a_2) \), then \( a_1 > a_2 \),
   - If \( H(a_1) = H(a_2) \), then \( a_1 = a_2 \).

### 3.2 Consistency in group decision-making

Group decision-making generally requires two or more decision-makers to find a solution to a decision problem. However, due to the different social experiences and personal cognitions of different decision-makers, the decisions they make tend to be biased. Therefore, in group decision-making, it is difficult for decision-makers to reach consensus and find a common solution; this problem is also a hot issue studied by researchers. Decision-makers’ preference for something can be expressed by q-ROFN. In this part, an interaction and feedback mechanism in a q-rung orthopair fuzzy information environment is proposed to help decision-makers obtain consistency in group decision-making.

**Definition 4.** (Liao et al.) For any two q-ROFNs \( \alpha = (u_\alpha, v_\alpha, \pi_\alpha), \beta = (u_\beta, v_\beta, \pi_\beta) \), the normalized Hamming distance between them is defined as follows:
\[
d(\alpha, \beta) = \frac{1}{2} \left( |u_\alpha - u_\beta| + |v_\alpha - v_\beta| + |\pi_\alpha - \pi_\beta| \right)
\]  
(1)

Where the Hamming distance satisfies \( 0 \leq d(\alpha, \beta) \leq 1 \) and indicates the difference between two q-ROFNs.

**Definition 5.** (Miao and Wu) For any two q-ROF preference relation matrices \( R_1 = \left( r_{ij}^1 \right)_{m \times n} \) and \( R_2 = \left( r_{ij}^2 \right)_{m \times n} \), the consistency measure between matrix \( R_1 \) and \( R_2 \) is defined as follows:
\[
C(R_1, R_2) = 1 - \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} d(r_{ij}^1, r_{ij}^2)
\]  
(2)

The larger \( C(R_1, R_2) \) is, the more similar and closer the matrix \( R_1 \) is to \( R_2 \).

**Definition 6.** (Miao and Wu) According to the definition of the consistency measure, the overall consistency level of decision-makers \( q \) is
\[ CC_k = \sum_{i=1}^{s} C(R_i, R_k) \]  

We believe that the greater the consistency of decision information between decision-makers \( e_k \) and other decision-makers, the greater the weight. The objective weight of the decision-maker \( e_k \) is:

\[ w_k = \frac{CC_k}{\sum_{i=1}^{s} CC_i} \]  

**Definition 7.** (Miao and Wu\(^3\)) The group matrix composed of decision-maker weights and the decision matrix is

\[ R = \left( \sum_{k=1}^{s} w_k r_k^{ij} \right)_{mn} \]  

The weight of the decision-maker is multiplied by each element in the evaluation matrix composed of q-ROFNs, and then each decision matrix is added to obtain the group decision matrix. **Definition 8.** (Miao and Wu\(^3\)) The consistency measure \( \theta_k (k = 1, 2, \ldots, s) \) between the evaluation matrix \( R_k = (r_k^{ij})_{mn} \) and the group matrix \( R = (r_{ij})_{mn} \) of each decision-maker is defined as

\[ \theta_k = C(R_k, R) = 1 - \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} d(r_k^{ij}, r_{ij}) \]  

The consistency between each decision matrix and the group decision matrix is calculated according to Formula (6). For those with small consistency, the corresponding decision-makers should appropriately adjust their evaluation values.

### 3.3 BM operator

**Definition 9.** (Bonferroni\(^4\)) Let \( s, t \geq 0 \) and \( a_k (k = 1, 2, \ldots, m) \) be a collection of nonnegative real numbers; then, the aggregation function:

\[ BM^{s,t}(a_1, a_2, \ldots, a_m) = \left( \frac{1}{m(m-1)} \sum_{i \neq j} a_i^{s} a_j^{t} \right)^{1/s+t} \]  

is called the BM operator.

### 3.4 The q-rung orthopair fuzzy Bonferroni mean operator

**Definition 10.** Suppose \( a_k = (u_k, v_k) (k = 1, 2, \ldots, m) \) is a collection of q-ROFNs, and \( s, t \geq 0, q \geq 1 \); then, the q-ROFBM operator can be defined as

\[ q - ROFBM^{s,t}(a_1, a_2, \ldots, a_m) = \left( \frac{1}{m(m-1)} \sum_{i \neq j}^{m} a_i^{s} a_j^{t} \right)^{1/s+t} \]  

**Theorem 1.** Suppose \( a_k = (u_k, v_k) (k = 1, 2, \ldots, m) \) is a collection of q-ROFNs, and \( s, t \geq 0, q \geq 1 \); then, the result aggregated from Definition 8 is still a q-ROFN, and even
\[ q - ROFBM^{+T}(a_1, a_2, \ldots, a_n) = \left( 1 - \prod_{i,j=1}^{n} (1 - q_i^r v_{ij}^r)^{1/(m-1)} \right) \left( 1 - \prod_{i,j=\bar{1}}^{n} (1 - (1 - v_{ij}^r)^{1 - v_{ij}^r})^{1/(m-1)} \right) \] 

3.5 The partitioned Bonferroni mean operator

**Definition 11.** (Dutta and Guha\(^4\)) For any \( r, s > 0 \) with \( r + s > 0 \) , and \( T = \{a_1, a_2, \ldots, a_m\} \) with \( a_k \geq 0 (k = 1, 2, \ldots, m) \) , which is partitioned into \( d \) distinct sorts \( P_1, P_2, \ldots, P_d \) , where \( \bigcup_{h=1}^{d} P_h = T \) , the partitioned BM aggregation operator of dimension \( m \) is a mapping PBM:

\[ PBM^{+T}(a_1, a_2, \ldots, a_m) = \frac{1}{d} \sum_{h=1}^{d} \left( \frac{1}{|P_h|} \sum_{j \in P_h} d_j \left( \frac{1}{|P_h| - 1} \sum_{j \not\in P_h} d_j \right) \right)^{1/\varepsilon} \] 

where \( |P_h| \) denotes the cardinality of \( P_h \) and \( d \) is the number of partitioned sorts.

3.6 The q-ROFPBM operator and q-ROFWPBM operator

In this section, the q-ROFPBM operator and q-ROFWPBM operator are proposed, and their properties are proven.

**Definition 12.** Let \( T = \{a_1, a_2, \ldots, a_m\} \) be a collection of q-ROFNs, which is partitioned into \( d \) distinct sorts \( P_1, P_2, \ldots, P_d \) , and \( \bigcup_{h=1}^{d} P_h = T \) . The q-ROFPBM operator of dimension \( m \) is a mapping q-ROFPBM:

\[ q - ROFPBM^{+T}(a_1, a_2, \ldots, a_m) = \frac{1}{d} \sum_{h=1}^{d} \left( \frac{1}{|P_h|} \sum_{j \in P_h} a_j \left( \frac{1}{|P_h| - 1} \sum_{j \not\in P_h} a_j \right) \right)^{1/\varepsilon} \] 

where \( |P_h| \) denotes the cardinality of \( P_h \) and \( d \) is the number of partitioned sorts. The operator can not only represent the interaction relationship between any two q-ROFNs but also represent the independence of q-ROFNs in different partitions.

**Theorem 2.** Suppose \( a_k = (u_k, v_k) (k = 1, 2, \ldots, m) \) is a collection of q-ROFNs, and \( s, t \geq 0 \) , \( q \geq 1 \) ; then, the result aggregated from Definition 12 is still a q-ROFN, and even
\[ q - \text{ROFPBM}^{st}(a_1, a_2, \ldots, a_n) = \left\{ \prod_{d=1}^{d} \left( 1 - \prod_{i,j \neq j}^{i} \left( 1 - w_{ij} a_i \right) \right)^{1/2} \right\}^{1/2} \]  

**Definition 13.** Let \( T = \{a_1, a_2, \ldots, a_n\} \) be a collection of q-ROFNs, which is partitioned into \( d \) distinct sorts \( P_1, P_2, \ldots, P_d \), and \( \sum_{d=1}^{d} P_d = T \), and \( w_i \) denote the weight of each argument \( a_i \), satisfying \( 0 \leq w_i \leq 1 \) and \( \sum_{i=1}^{n} w_i = 1 \). For any \( s, t \geq 0 \) and \( s + t > 0 \),

\[ q - \text{ROFPBM}^{qs}(a_1, a_2, \ldots, a_n) = \frac{1}{d} \left\{ \sum_{d=1}^{d} \left( \frac{1}{P_d} \sum_{i,j \neq j}^{i} \left( w_{ij} a_i \right) \right) \right\}^{1/2} - \frac{1}{d} \left\{ \sum_{d=1}^{d} \left( \frac{1}{P_d} \sum_{i,j \neq j}^{i} \left( w_{ij} a_i \right) \right) \right\}^{1/2} \]

The operator adds a weight coefficient based on the q-ROFPBM operator, which can reflect the preference and attention of decision-makers for different criteria.

**Theorem 3.** Suppose \( a_k = (u_k, v_k) (k = 1, 2, \ldots, m) \) is a collection of q-ROFNs, and \( s, t \geq 0, q \geq 1 \); then, the result aggregated from Definition 13 is still a q-ROFN, and even

\[ q - \text{ROFPBM}^{qs}(a_1, a_2, \ldots, a_n) = \left\{ \prod_{d=1}^{d} \left( 1 - \prod_{i,j \neq j}^{i} \left( 1 - w_{ij} a_i \right) \right) \right\}^{1/2} \]

**Proof.** Based on the algorithm in Definition 2, we can get

\[ a_{s,t} = \left( u_{s,t}, \left( 1 - \left( 1 - v_{s,t} \right)^{1/q} \right)^{1/q} \right), \quad a_{s,t} = \left( u_{s,t}, \left( 1 - \left( 1 - v_{s,t} \right)^{1/q} \right)^{1/q} \right) \]

According to the number multiplication and product algorithms, we can obtain

\[ w_{s,t} a_{s,t} = \left( 1 - \left( 1 - u_{s,t} \right)^{1/q} \right)^{1/q}, \quad w_{s,t} a_{s,t} = \left( 1 - \left( 1 - u_{s,t} \right)^{1/q} \right)^{1/q} \]
\[(w_i a_i')(w_j a_j') = \left[ \left( 1 - (1 - u_i^a_i')^{\nu_i} \right) \left( 1 - (1 - v_j^a_i')^{\nu_j} \right) \right]^{\frac{1}{\nu_i}} \left[ \left( 1 - (1 - u_j^a_i')^{\nu_j} \right) \left( 1 - (1 - v_i^a_i')^{\nu_i} \right) \right]^{\frac{1}{\nu_j}} \]

According to mathematical induction, we can obtain

\[
\sum_{i,j} (w_i a_i')(w_j a_j') = \left[ \prod_{i,j} \left( 1 - (1 - u_i^a_i')^{\nu_i} \right) \left( 1 - (1 - v_j^a_i')^{\nu_j} \right) \right]^{\frac{1}{\nu_i}} \left[ \prod_{i,j} \left( 1 - (1 - u_j^a_i')^{\nu_j} \right) \left( 1 - (1 - v_i^a_i')^{\nu_i} \right) \right]^{\frac{1}{\nu_j}}
\]

Then,

\[
\frac{1}{|P_a|(|P_b| - 1)} \sum_{i,j} (w_i a_i')(w_j a_j') = \left[ \prod_{i,j} \left( 1 - (1 - u_i^a_i')^{\nu_i} \right) \left( 1 - (1 - v_j^a_i')^{\nu_j} \right) \right]^{\frac{1}{\nu_i}} \left[ \prod_{i,j} \left( 1 - (1 - u_j^a_i')^{\nu_j} \right) \left( 1 - (1 - v_i^a_i')^{\nu_i} \right) \right]^{\frac{1}{\nu_j}}
\]

Then,

\[
\left( \frac{1}{|P_a|(|P_b| - 1)} \sum_{i,j} (w_i a_i')(w_j a_j') \right)^{\frac{1}{\nu_i}} \left( \frac{1}{|P_a|(|P_b| - 1)} \sum_{i,j} (w_i a_i')(w_j a_j') \right)^{\frac{1}{\nu_j}}
\]

According to mathematical induction, we can obtain

\[
\sum_{i=1}^d \left( \frac{1}{|P_a|(|P_b| - 1)} \sum_{i,j} (w_i a_i')(w_j a_j') \right)^{\frac{1}{\nu_i}} = \left[ \prod_{i=1}^d \left( 1 - \prod_{i,j} \left( 1 - (1 - u_i^a_i')^{\nu_i} \right) \left( 1 - (1 - v_j^a_i')^{\nu_j} \right) \right]^{\frac{1}{\nu_i}} \left[ \prod_{i=1}^d \left( 1 - \prod_{i,j} \left( 1 - (1 - u_j^a_i')^{\nu_j} \right) \left( 1 - (1 - v_i^a_i')^{\nu_i} \right) \right]^{\frac{1}{\nu_j}}
\]

According to the number multiplication algorithm, we can obtain
\[ q - ROFWPM^{(t)}(a_1, a_2, \ldots, a_n) = 1 - \frac{1}{d} \sum_{k=1}^{d} \left( \frac{1}{P_k} - 1 \right) \sum_{i,j \in P_k} (w_i' a_i^*) (w_j' a_j^*) \right) \] 

\[ = \left\{ 1 - \prod_{h=1}^{d} \left( 1 - \left( 1 - \left( 1 - \left( 1 - u^w \right)^{w'} \right) \left( 1 - v^w \right)^{v'} \right) \right) \right\} \] 

**Theorem 4.** (Idempotency) If all \( a_j = (u_j, v_j) j = (1, 2, \ldots, n) \) are similar to \( a = (u, v) \), then

\[ q - ROFWPM(a_1, a_2, \ldots, a_n) = a \] 

Proof.

Let \( s = t = 1 \), \( w_i = w_j = w \), since \( a_j = a = (u, v) \), then

\[ 1 - \prod_{h=1}^{d} \left( 1 - \left( 1 - \left( 1 - u^w \right)^{w'} \right) \left( 1 - v^w \right)^{v'} \right) \right\} \] 

\[ = \left\{ 1 - \prod_{h=1}^{d} \left( 1 - \left( 1 - u^w \right)^{w} \right) \right\} \] 

\[ = \left\{ 1 - \prod_{h=1}^{d} \left( 1 - \left( 1 - u^w \right)^{w} \right) \right\} \] 

\[ = \left\{ 1 - \prod_{h=1}^{d} \left( 1 - \left( 1 - u^w \right)^{w} \right) \right\} \] 

\[ = \left\{ 1 - \prod_{h=1}^{d} \left( 1 - \left( 1 - u^w \right)^{w} \right) \right\} \] 

\[ = \left\{ 1 - \prod_{h=1}^{d} \left( 1 - \left( 1 - u^w \right)^{w} \right) \right\} \] 

\[ = 1 - (1 - u^w)^{w} = (u^w)^{w} = u \]
Therefore,

\[
q - \text{ROFWPM}^{\tau\tau}(a_1, a__2, \ldots, a_n) =
\begin{pmatrix}
1 - \prod_{d=1}^{d} \left(1 - \prod_{i,j \neq i}^{n} \left(1 - (1 - v^{\prime\prime}_{i,j})^{\tau\tau}\right) \left(1 - (1 - u^{\prime\prime}_{i,j})^{\tau\tau}\right) \right)
\end{pmatrix}
\]

\[
= (u, v) = a
\]

Theorem 4 is correct.

**Theorem 5.** (Commutativity) Let \(a_j = (u_j, v_j)\) be one q-ROFS; if \(a_j^{\tau}\) is the permutation of \(a_j\) and \(a_j^{\prime} = (u^{\prime}, v^{\prime})\), then
\( q - ROFWPBM^+(a_1, a_2, \ldots, a_m) = q - ROFWPBM^+(a'_1, a'_2, \ldots, a'_m) \)

Proof.

Since

\[
q - ROFWPBM^+(a_1, a_2, \ldots, a_m) = \left\{ 1 - \prod_{k=1}^{d} \left[ 1 - \prod_{i \neq j} \left( 1 - \left( 1 - (u_i)^{y_i} \right)^{\nu_i} \right) \left( 1 - \left( 1 - (v_j)^{y_j} \right)^{\nu_j} \right) \right]^{\frac{1}{\nu_i y_i}} \right\}^{\frac{1}{\nu_i y_i}}.
\]

Then,

\[
q - ROFWPBM^+(a'_1, a'_2, \ldots, a'_m) = \left\{ 1 - \prod_{k=1}^{d} \left[ 1 - \prod_{i \neq j} \left( 1 - \left( 1 - (u'_i)^{y_i} \right)^{\nu_i} \right) \left( 1 - \left( 1 - (v'_j)^{y_j} \right)^{\nu_j} \right) \right]^{\frac{1}{\nu_i y_i}} \right\}^{\frac{1}{\nu_i y_i}}.
\]

Since \( a'_j \) is the permutation of \( a_j \), we have \( a'_j = a_j \) for all \( j = 1, 2, \ldots, n \). Then we can obtain:

\( q - ROFWPBM^+(a_1, a_2, \ldots, a_m) = q - ROFWPBM^+(a'_1, a'_2, \ldots, a'_m) \)

Theorem 6. (Boundedness) For a collection of \( q \)-ROFNs, \( a_j = (u_j, v_j)(j = 1, 2, \ldots, n) \),

\( a^+ = (\max_j u_j, \min_j v_j), a^- = (\min_j u_j, \max_j v_j) \),

Then, \( a^- < q - ROFWPBM^+(a_1, a_2, \ldots, a_m) < a^+ \).

We can use a method similar to Theorem 3 and Theorem 4 to prove that Theorem 5 is correct, and the specific proof details are omitted here.

4. The group decision-making algorithm based on the IFM and \( q \)-ROFWPBM operator

In this section, a group decision-making method based on the IFM and \( q \)-ROFWPBM operator is proposed, and the main steps of this method are introduced. The decision-making process is shown in Fig. 1:
The decision-maker scores and obtains the evaluation information matrix

Calculate decision maker weights

The group decision matrix $R$ is calculated

Calculate the consistency $\theta_k$ between each evaluation matrix and group decision matrix

Decision makers adjust evaluation information

\[ \theta_k > \zeta \]

Aggregate the group decision matrix $R$ with the $q$-ROFWPBM operator

Calculate the candidate score and select the best candidate

**Fig. 1.** The framework of the decision-making process

Set $A=(A_1, A_2, \ldots, A_m)$ as $m$ alternatives, $C=(C_1, C_2, \ldots, C_m)$ as $n$ criteria for each variable, and set $w = (w_1, w_2, \ldots, w_n)^T$ as the weight corresponding to each evaluation criterion, satisfying $\sum_{j=1}^{n} w_j = 1$ and $0 \leq w_j \leq 1$. For the evaluation criteria $C_j$ of alternative $A_i$, the evaluation information can be expressed as $q$-ROF $a = \{u, v\}$.

**Step 1.** There are $s$ experts, defined as $e_1, e_2, \ldots, e_s$, and each expert scores the criterion $a_j$ of the scheme $x_i$ to obtain a matrix $r_{ij}^k(\mu_j, \nu_j, \pi_j)$ with the $q$-ROF form, $k = 1, 2, \ldots, s$, and obtains a total matrix $R = (r_{ij}^k)_{s \times m}$, $k = 1, 2, \ldots, s$.

**Step 2.** Determine the overall weight $w_k$ of decision-maker $e_i$ according to Equations (2), (3), and (4).

**Step 3.** The group decision matrix $R = (r_{ij}^k)_{s \times m}$ is determined by Equation (5).

**Step 4.** The consistency measure $\theta_k (k = 1, 2, \ldots, s)$ of each decision-maker is determined by Equation (6).
Step 5. Comparing $\theta_i$ with $\xi$ (consistency threshold), if $\theta_i > \xi$, go to Step 7; if $\theta_i < \xi$, go to Step 6.

Step 6. Taking the decision matrix of the decision-maker with the greatest consistency as a reference, decision-maker $e_i$ adjusts his evaluation value to a new decision matrix $R_{kr}^{(l)} = (r_{ki}^{(l)})_{mn}$, $(l = 1, 2, ...)$ and returns to Step 2 (recalculate the decision-maker weight and group decision matrix).

Step 7. For the final group decision matrix $R = (r_{ki})_{mn}$, the information is aggregated by Equation (12).

Step 8. For the final aggregation results, the score function is used to calculate the final score of each scheme and obtain the final ranking of the scheme.

5. Numerical analysis

5.1 Problem description

Supplier selection is an important part of the comprehensive ability of modern enterprises. With the continuous development of enterprise scale, supplier selection has become a key decision-making task. In different stages of enterprise development, the requirements for suppliers are different. For example, in the early stage of enterprise development, the requirements for supply cost and product quality are higher, while in the mature stage of enterprise development, the efficiency of supply and the ability to deal with emergencies are more valued. Taking Hongxing Erke as an example to consider the dynamic changes in enterprise demand for suppliers and to help enterprises select the most suitable suppliers, the decision algorithm proposed in this study is applied to evaluate suppliers. We selected four suppliers of Hongxing Erke as candidate schemes. According to the actual situation and combined with the relevant research on existing supplier selection, this study selected product quality, transportation efficiency, cost, and flexibility as the four evaluation indicators, as shown in Table 1.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Description</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality (C1)</td>
<td>The supplier must ensure high product quality</td>
<td>[18], [15], [75], [76]</td>
</tr>
<tr>
<td>Delivery (C2)</td>
<td>Supplier's efficiency in delivering products</td>
<td>[18], [75]</td>
</tr>
<tr>
<td>Cost (C3)</td>
<td>Supplier’s production cost and supply cost</td>
<td>[18], [15], [76]</td>
</tr>
<tr>
<td>Flexibility (C4)</td>
<td>Suppliers should have enough flexibility to deal with emergencies</td>
<td>[4]</td>
</tr>
</tbody>
</table>

5.2 Decision procedure

Step 1. Enterprise development is divided into three stages: early stage, middle stage, and late stage. Some experienced entrepreneurs or businesspersons are invited to be divided into three decision-making groups $e_1, e_2, e_3$. The evaluation values in the form of q-ROFN are given for the four supplier evaluation indices in the three development stages to form three decision matrices $R_1, R_2, R_3$, as shown in Table 2-Table 4:

<table>
<thead>
<tr>
<th>C11</th>
<th>C12</th>
<th>C13</th>
<th>C14</th>
<th>C21</th>
<th>C22</th>
<th>C23</th>
<th>C24</th>
<th>C31</th>
<th>C32</th>
<th>C33</th>
<th>C34</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.8,0.1</td>
<td>0.6,0.4</td>
<td>0.6,0.4</td>
<td>0.9,0.2</td>
<td>0.6,0.5</td>
<td>0.8,0.2</td>
<td>0.8,0.2</td>
<td>0.9,0.2</td>
<td>0.8,0.3</td>
<td>0.7,0.2</td>
<td>0.7,0.3</td>
</tr>
<tr>
<td>A2</td>
<td>0.7,0.3</td>
<td>0.7,0.4</td>
<td>0.7,0.4</td>
<td>0.9,0.1</td>
<td>0.7,0.2</td>
<td>0.8,0.3</td>
<td>0.8,0.2</td>
<td>0.9,0.1</td>
<td>0.8,0.2</td>
<td>0.8,0.2</td>
<td>0.8,0.2</td>
</tr>
</tbody>
</table>
The consistency level of the third decision matrix is calculated by Formula (2):

\[ C(R_3, R_3) = 0.625334 \]

Formula (3) is used to determine the overall consistency level of each group of decision-makers as follows:

\[ CC_1 = 1.269228, CC_2 = 1.277607, CC_3 = 1.242288 \]

Calculate the weight of each group of decision-makers by using Formula (4): \( w_1 = 0.334966 \), \( w_2 = 0.678813 \), \( w_3 = 0.616955 \).

Step 3. The group decision matrix \( M_i \) is formed by combining the weight of each group of decision-makers with their respective decision matrix through Formula (5), as shown in Table 5:

\[ M_1 = \begin{pmatrix}
A_3 & 0.70.4 & 0.80.3 & 0.80.2 & 0.80.2 & 0.80.2 & 0.80.3 & 0.80.3 & 0.80.3 & 0.80.2 & 0.80.2 & 0.80.3 \\
A_4 & 0.90.2 & 0.90.2 & 0.80.2 & 0.80.2 & 0.80.2 & 0.80.2 & 0.80.2 & 0.80.2 & 0.80.2 & 0.80.2 & 0.80.2 \\
\end{pmatrix} \]

Step 4. The consistency measures of each group decision matrix and group decision matrix \( M_i \) are determined by Formula (6): \( \theta_1(1) = 0.707885 \), \( \theta_2(1) = 0.72495 \), \( \theta_3(1) = 0.678813 \).

Step 5. We set the consistency threshold to \( \xi = 0.7 \), then \( \theta_i < \xi \). The consistency level of the third group of decision-makers is low, so enter Step 6.

Step 6. \( \theta_2(1) > \theta_1(1) > \theta_3(1) \), the consistency level of the second group of decision-makers is the highest, so we use the second decision matrix as a reference to adjust the evaluation information in the third matrix that does not meet the consistency requirements. The adjusted evaluation information is shown in Table 6:

\[ R_{2(1)} = \begin{pmatrix}
A_3 & 0.70.4 & 0.80.3 & 0.80.2 & 0.80.2 & 0.80.3 & 0.70.3 & 0.70.4 & 0.70.4 & 0.70.4 \\
A_4 & 0.90.2 & 0.90.2 & 0.80.2 & 0.80.2 & 0.90.2 & 0.90.2 & 0.90.2 & 0.90.2 & 0.90.2 \\
\end{pmatrix} \]
Then, we return to Step 2, and the weights of each group of recalculated decision-makers are $w_1 = 0.321082$, $w_2 = 0.342795$, and $w_3 = 0.336123$. The new group decision matrix $M_1$ obtained according to the new weight and evaluation matrix is shown in Table 7.

**Table 7. Group decision matrix $M_2$**

<table>
<thead>
<tr>
<th></th>
<th>Early Stage</th>
<th>Middle Stage</th>
<th>Late Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.802</td>
<td>0.802</td>
<td>0.803</td>
</tr>
<tr>
<td>C2</td>
<td>0.703</td>
<td>0.802</td>
<td>0.802</td>
</tr>
<tr>
<td>C3</td>
<td>0.704</td>
<td>0.801</td>
<td>0.703</td>
</tr>
<tr>
<td>C4</td>
<td>0.801</td>
<td>0.802</td>
<td>0.703</td>
</tr>
</tbody>
</table>

Then, calculate the consistency measure between each group decision matrix and group decision matrix $M_2$ through Formula (6): $\theta_{1(2)} = 0.695083$, $\theta_{2(2)} = 0.75429$, $\theta_{3(2)} = 0.729591$, $\theta_{1(2)} < \xi$; therefore, the matrix $R_i$ with the highest consistency level is used as a reference to adjust the information in the matrix $R_i$ to obtain a new matrix $R_{i(2)}$, as shown in Table 8:

**Table 8. Assessment matrix $R_{i(2)}$**

<table>
<thead>
<tr>
<th></th>
<th>Early Stage</th>
<th>Middle Stage</th>
<th>Late Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.803</td>
<td>0.803</td>
<td>0.604</td>
</tr>
<tr>
<td>C2</td>
<td>0.703</td>
<td>0.802</td>
<td>0.801</td>
</tr>
<tr>
<td>C3</td>
<td>0.704</td>
<td>0.803</td>
<td>0.802</td>
</tr>
<tr>
<td>C4</td>
<td>0.802</td>
<td>0.703</td>
<td>0.604</td>
</tr>
</tbody>
</table>

Then, we return to Step 2, and the weights of each group of recalculated decision-makers are $w_1 = 0.326671$, $w_2 = 0.340476$, and $w_3 = 0.332853$. The new group decision matrix $M_i$ obtained according to the new weight and evaluation matrix is shown in Table 9.

**Table 9. Group decision matrix $M_3$**

<table>
<thead>
<tr>
<th></th>
<th>Early Stage</th>
<th>Middle Stage</th>
<th>Late Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.802</td>
<td>0.803</td>
<td>0.704</td>
</tr>
<tr>
<td>C2</td>
<td>0.802</td>
<td>0.803</td>
<td>0.802</td>
</tr>
<tr>
<td>C3</td>
<td>0.803</td>
<td>0.704</td>
<td>0.803</td>
</tr>
<tr>
<td>C4</td>
<td>0.802</td>
<td>0.803</td>
<td>0.802</td>
</tr>
</tbody>
</table>

Then, calculate the consistency measures of each decision matrix and group decision matrix $M_i$ through Formula (6): $\theta_{1(3)} = 0.718712$, $\theta_{2(3)} = 0.766033$, $\theta_{3(3)} = 0.724818$, satisfies $\theta_{i} > \xi$. At this time, the three groups of decision information matrices meet the consistency level requirements, so we think $M_3$ is the final group decision matrix for the next step.

Step 7. The q-ROFWPBM operator proposed in this paper is used to aggregate the information of the matrix $M_i$; here, we take $q = 2, s = t = 1$. In this paper, the enterprise development stage is divided into three stages: early stage, middle stage, and late stage, with the weights of $w_1 = 0.35, w_2 = 0.3$, and $w_3 = 0.35$. Different weights are given to the four attributes of each stage, as shown in Table 10:
Table 10. Attribute weight of each stage

<table>
<thead>
<tr>
<th></th>
<th>Early Stage</th>
<th>Middle Stage</th>
<th>Late Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>C11</td>
<td>0.11</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>C12</td>
<td>0.08</td>
<td>0.1</td>
<td>0.07</td>
</tr>
<tr>
<td>C13</td>
<td>0.07</td>
<td>0.078</td>
<td>0.074</td>
</tr>
<tr>
<td>C14</td>
<td>0.078</td>
<td>0.097</td>
<td>0.07</td>
</tr>
<tr>
<td>C21</td>
<td>0.078</td>
<td>0.1</td>
<td>0.073</td>
</tr>
<tr>
<td>C22</td>
<td>0.097</td>
<td>0.1</td>
<td>0.08</td>
</tr>
<tr>
<td>C23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C32</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C34</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The final evaluation results of each supplier are aggregated by the q-ROFWPBM operator, and the score of each supplier is calculated by the score function, as shown in Table 11:

Table 11. Final evaluation value and score of each supplier

<table>
<thead>
<tr>
<th>Evaluation Value</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 &lt;0.23,0.32&gt;</td>
<td>-0.045</td>
</tr>
<tr>
<td>A2 &lt;0.25,0.30&gt;</td>
<td>-0.029</td>
</tr>
<tr>
<td>A3 &lt;0.24,0.32&gt;</td>
<td>-0.042</td>
</tr>
<tr>
<td>A4 &lt;0.21,0.34&gt;</td>
<td>-0.074</td>
</tr>
</tbody>
</table>

According to the final score, the final ranking of the four suppliers is A2 > A3 > A1 > A4. A2 is the best supplier.

5.3. Sensitivity analysis

In this section, we analyze the influence of parameter q and the changes in parameters s and t on the decision results in the q-ROFWPBM operator. First, we analyze the impact of the change in parameter q on the decision results. Here, we keep q = 2 unchanged and change q from 2 to 8. The supplier scores and ranking results are shown in Table 12:

Table 12. Alternative ranking of different parameter q in q-ROFWPBM operator

<table>
<thead>
<tr>
<th>q</th>
<th>Score</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>q=2</td>
<td>S(A1)=-0.045; S(A2)=-0.029</td>
<td>A2 &gt; A3 &gt; A1 &gt; A4</td>
</tr>
<tr>
<td></td>
<td>S(A3)=-0.042; S(A4)=-0.074</td>
<td></td>
</tr>
<tr>
<td>q=3</td>
<td>S(A1)=-0.024; S(A2)=-0.011</td>
<td>A2 &gt; A3 &gt; A1 &gt; A4</td>
</tr>
<tr>
<td></td>
<td>S(A3)=-0.021; S(A4)=-0.048</td>
<td></td>
</tr>
<tr>
<td>q=4</td>
<td>S(A1)=-0.013 S(A2)=-0.003</td>
<td>A2 &gt; A3 &gt; A1 &gt; A4</td>
</tr>
<tr>
<td></td>
<td>S(A3)=-0.011; S(A4)=-0.034</td>
<td></td>
</tr>
<tr>
<td>q=5</td>
<td>S(A1)=-0.008; S(A2)=-0.001</td>
<td>A2 &gt; A3 &gt; A1 &gt; A4</td>
</tr>
<tr>
<td></td>
<td>S(A3)=-0.006; S(A4)=-0.024</td>
<td></td>
</tr>
<tr>
<td>q=6</td>
<td>S(A1)=-0.004; S(A2)=-0.003</td>
<td>A2 &gt; A3 &gt; A1 &gt; A4</td>
</tr>
<tr>
<td></td>
<td>S(A3)=-0.003; S(A4)=-0.018</td>
<td></td>
</tr>
<tr>
<td>q=7</td>
<td>S(A1)=-0.003; S(A2)=-0.003</td>
<td>A2 &gt; A3 &gt; A1 &gt; A4</td>
</tr>
<tr>
<td></td>
<td>S(A3)=-0.001; S(A4)=-0.014</td>
<td></td>
</tr>
<tr>
<td>q=8</td>
<td>S(A1)=-0.001; S(A2)=-0.003</td>
<td>A2 &gt; A3 &gt; A1 &gt; A4</td>
</tr>
<tr>
<td></td>
<td>S(A3)=-0.000; S(A4)=-0.011</td>
<td></td>
</tr>
</tbody>
</table>

According to the results in the table, the score of each supplier changes with the change of q from 2 to 8, but the final ranking order does not change. In this method, the value of q represents the complexity of the decision environment and conditions. The greater q is, the more complex the decision conditions are. In the actual decision environment, the decision-maker can change the value of q according to the actual situation and needs to obtain satisfactory decision results.

Next, we keep q = 2 unchanged and change the values of s and t. The supplier scores and ranking results are shown in Table 13:

Table 13. Alternative ranking of different parameters s and t in the q-ROFWPBM operator

<table>
<thead>
<tr>
<th>s,t</th>
<th>Score</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>s=t=2</td>
<td>S(A1)=-0.097; S(A2)=-0.086</td>
<td>A2 &gt; A3 &gt; A1 &gt; A4</td>
</tr>
<tr>
<td></td>
<td>S(A3)=-0.096; S(A4)=-0.121</td>
<td></td>
</tr>
<tr>
<td>s=t=3</td>
<td>S(A1)=-0.115; S(A2)=-0.106</td>
<td>A2 &gt; A3 &gt; A1 &gt; A4</td>
</tr>
<tr>
<td></td>
<td>S(A3)=-0.114; S(A4)=-0.135</td>
<td></td>
</tr>
</tbody>
</table>
[In the state of the evaluation criteria. Method 6 combines the IFM operator proposed in this paper with the ROFWPBM operator to aggregate the evaluation matrices of different decision methods and the results of decision making environment. In the actual decision-making, the decision-maker can adjust the values of the parameters according to the situation to obtain more accurate decision-making results.

5.4 Comparative analysis

To verify the effectiveness of the method proposed in this research, we select some methods from other studies to calculate the same number of examples above and compare the characteristics of different methods and the results of decision-making, including the method based on the q-ROFWA operator and q-ROFWG operator in the literature [70], the method based on the q-rung orthopair fuzzy weighted Bonferroni mean (q-ROFWBM) operator proposed in the literature [37], method 5 combining the q-ROFWA operator and q-ROFWPBM operator, in which the q-ROFWA operator is used to aggregate the evaluation matrices of different decision-makers, and the q-ROFWPBM operator is used to aggregate the evaluation criteria. Method 6 combines the IFM proposed in this paper with the q-ROFWBM operator, and method 7 combines the IFM with the q-ROFWA operator. In methods 2-5, the weight of each decision-maker is set to 1/3.

We use the above methods to calculate the same evaluation data, and the scores and rankings of the schemes obtained by each method are shown in Table 14:

<table>
<thead>
<tr>
<th>Number</th>
<th>Methods</th>
<th>Score</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Proposed method</td>
<td>S(A1)=0.045; S(A2)=0.029</td>
<td>A2&gt; A3&gt; A1&gt; A4</td>
</tr>
<tr>
<td>2</td>
<td>The q-ROFWA operator in [70]</td>
<td>S(A1)=0.554; S(A2)=0.589</td>
<td>A2&gt; A3&gt; A1&gt; A4</td>
</tr>
<tr>
<td>3</td>
<td>The q-ROFWG operator in [70]</td>
<td>S(A1)=0.586; S(A2)=0.504</td>
<td>A3&gt; A2&gt; A1&gt; A4</td>
</tr>
<tr>
<td>4</td>
<td>The q-ROFWBM operator in [37]</td>
<td>S(A1)=0.611; S(A2)=0.591</td>
<td>A2&gt; A3&gt; A1&gt; A4</td>
</tr>
<tr>
<td>5</td>
<td>q-ROFWA operator+q-ROFWPBM operator</td>
<td>S(A1)=0.7276; S(A2)=0.7040</td>
<td>A3&gt; A2&gt; A1&gt; A4</td>
</tr>
<tr>
<td>6</td>
<td>IFM+q-ROFWBM operator (q=2,s=t=1)</td>
<td>S(A1)=0.7276; S(A2)=0.7040</td>
<td>A2&gt; A1&gt; A3&gt; A4</td>
</tr>
<tr>
<td>7</td>
<td>IFM+q-ROFWA operator</td>
<td>S(A1)=0.572; S(A2)=0.622</td>
<td>A2&gt; A3&gt; A1&gt; A4</td>
</tr>
</tbody>
</table>

From the results in Table 14, the scheme scores obtained by different methods are different,
and the ranking results of the schemes are also changed. The best supplier calculated by methods 2, 4, 6, and 7 is A2, which is the same result obtained by the method proposed in this paper. However, there are some differences in the scheme ranking; the best supplier calculated by methods 3 and 5 is A3. Methods 2-5 do not adopt the group decision-making mechanism based on interaction and feedback, do not consider the consistency of opinions among different decision-making groups, and do not deal with the evaluation values with large differences, but directly aggregate the initial evaluation information, and the final results may have some errors. Methods 6 and 7 adopt the group decision-making mechanism based on interaction and feedback without using the partitioned operator. The results are also different from those obtained by using the method in this paper. The difference is that the partitioned operator is not applied and the independence between different stages of enterprise development is ignored. Method 7 uses the q-ROFWA operator instead of the q-ROFWPBM operator, without considering the interaction between any two evaluation criteria and the independence between different stages of enterprise development. The difference between method 5 and the method proposed in this paper is that instead of using the group decision-making mechanism based on interaction and feedback, the q-ROFWA operator is selected to aggregate the evaluation matrix of different decision-makers, ignoring the weight of different decision-makers and the difference of opinions between them. Methods 2 and 3 do not take into account the relationship between the evaluation objects, but aggregate all the evaluation objects, and do not take into account the interaction between any two evaluation criteria and the independence of evaluation criteria in different development stages.

To verify the robustness of the proposed method, we add some outliers to the original evaluation data, including the evaluation value of 0 and the evaluation value with a large difference, to test whether the proposed method remains stable under uncertain factors. Next, the evaluation data with outliers are calculated by the methods in Table 14. The evaluation matrix and the group decision-making process of interaction and feedback are shown in Appendix 1. The scheme scores and rankings are shown in Table 15:

Table 15. Alternative ranking by various approaches with outliers

<table>
<thead>
<tr>
<th>Number</th>
<th>Methods</th>
<th>Score</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Proposed method</td>
<td>S(A1)=0.059; S(A2)=0.050; S(A3)=0.058; S(A4)=-0.069</td>
<td>A2&gt;A3&gt;A1&gt;A4</td>
</tr>
<tr>
<td>2</td>
<td>The q-ROFWA operator in [70]</td>
<td>S(A1)=0.592; S(A2)=0.590; S(A3)=0.609; S(A4)=-0.477</td>
<td>A3&gt;A1&gt;A2&gt;A4</td>
</tr>
<tr>
<td>3</td>
<td>The q-ROFWG operator in [70]</td>
<td>S(A1)=0.415; S(A2)=0.064; S(A3)=-0.083; S(A4)=-0.096</td>
<td>A1&gt;A2&gt;A3&gt;A4</td>
</tr>
<tr>
<td>4</td>
<td>The q-ROFWBM operator in [37] (q=2, s=t=1)</td>
<td>S(A1)=0.608; S(A2)=0.585; S(A3)=-0.617; S(A4)=-0.655</td>
<td>A2&gt;A1&gt;A3&gt;A4</td>
</tr>
<tr>
<td>5</td>
<td>q-ROFWA operator+q-ROFWB operator</td>
<td>S(A1)=-0.946; S(A2)=-0.947; S(A3)=-0.943; S(A4)=-0.949</td>
<td>A3&gt;A1&gt;A2&gt;A4</td>
</tr>
<tr>
<td>6</td>
<td>IFM+q-ROFWB operator (q=2, s=t=1)</td>
<td>S(A1)=-0.748; S(A2)=-0.735; S(A3)=-0.749; S(A4)=-0.763</td>
<td>A2&gt;A1&gt;A3&gt;A4</td>
</tr>
<tr>
<td>7</td>
<td>IFM+q-ROFWA operator</td>
<td>S(A1)=0.528; S(A2)=0.547; S(A3)=0.540; S(A4)=0.486</td>
<td>A2&gt;A3&gt;A1&gt;A4</td>
</tr>
</tbody>
</table>

By comparing the results in Table 15 with those in Table 14, we find that after adding outliers, the sorting results of methods 2-5 have changed to varying degrees compared with those without outliers, while the scheme sorting results obtained by methods 6, method 7, and the method proposed in this paper have not changed compared with those before adding outliers, because these methods adopt the group decision-making mechanism based on interaction and feedback, which can effectively deal with outliers and the large differences in the evaluation values of different decision-
makers. However, other methods still directly aggregate the original evaluation matrix and do not deal with outliers, so the ranking results change greatly compared with the previous ones. This shows that the group decision-making method proposed in this paper will not be greatly affected by the interference of uncertain factors and has good robustness.

6. Conclusion

The problem of supplier selection has an extremely important impact on the development of an enterprise. The selection of a suitable supplier for an enterprise can be regarded as an extremely important multicriteria group decision-making problem. To comprehensively consider the dynamic variability of the conditions for enterprises to select suppliers in different development stages and the interaction between multiple evaluation criteria, this paper proposes the q-ROFWPBM operator, which combines the IFM of group decision-making to construct a multiattribute group decision-making framework and applies it to the Hongxing Erke supplier selection problem for verification. The results show that the method proposed in this paper can not only solve the problem of large differences in opinions between different decision-makers in the group decision-making process but also consider the dynamic variability of the enterprise’s demand for suppliers at different stages of development, providing theoretical support for the enterprise's dynamic supplier selection problem. The main contributions of this article are as follows: This paper uses q-ROFs to solve the problem of characterizing evaluation information in complex environments, which is conducive to improving the accuracy of decision-making results in the context of complex decision-making.

- The IFM constructed in this paper can effectively solve the problem of large differences in the evaluation information of individual decision-makers in group decision-making problems and improve the consistency of group decision-making and the accuracy of decision-making results.
- The operator proposed in this paper can not only represent the interaction relationship between some decision objects and consider the dynamic change of evaluation criteria but also represent the relationship between any two evaluation criteria, fully consider the complexity and variability of information in the decision-making process, and provide decision support for solving dynamic supplier selection and other similar problems.

Based on the shortcomings of this paper and existing research, future research can also be carried out from the following aspects:

- Regarding the consistency of group decision-making, other more effective and flexible methods can be considered to help improve the scientificity and accuracy of decision-making.
- The IFM and the idea of partitioning proposed in this paper can be combined with other fuzzy sets, such as T-spherical fuzzy sets, to develop other operators, which can adapt to more complex decision-making environments and conditions and enhance the flexibility of the method application.
- In this paper, the proposed method is applied to the enterprise supplier selection problem. In the future, we can try to apply this method to group decision-making problems in other fields, solve more problems, and expand the application scope of this method.

Conflict of Interest Statement
We declare that there are no conflicts of interest regarding the publication of this paper

Appendix 1
Step 1. First, we add a few outliers to the original evaluation matrices to obtain new evaluation matrices $R'_1, R'_2, R'_3$, as shown in Table 16–Table 18.

**Table 16. Assessment matrix $R'_1$**

<table>
<thead>
<tr>
<th></th>
<th>Early Stage</th>
<th>Middle Stage</th>
<th>Late Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C11</td>
<td>C12</td>
<td>C13</td>
</tr>
<tr>
<td>A1</td>
<td>0.8,0.3</td>
<td>0.8,0.2</td>
<td>0.6,0.4</td>
</tr>
<tr>
<td>A2</td>
<td>0.7,0.3</td>
<td>0.7,0.4</td>
<td>0.7,0.4</td>
</tr>
<tr>
<td>A3</td>
<td>0.7,0.4</td>
<td>0.8,0.3</td>
<td>0.8,0.2</td>
</tr>
<tr>
<td>A4</td>
<td>0.9,0.2</td>
<td>0.9,0.2</td>
<td>0.8,0.2</td>
</tr>
</tbody>
</table>

**Table 17. Assessment matrix $R'_2$**

<table>
<thead>
<tr>
<th></th>
<th>Early Stage</th>
<th>Middle Stage</th>
<th>Late Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C11</td>
<td>C12</td>
<td>C13</td>
</tr>
<tr>
<td>A1</td>
<td>0.8,0.2</td>
<td>0.8,0.2</td>
<td>0.7,0.2</td>
</tr>
<tr>
<td>A2</td>
<td>0.8,0.2</td>
<td>0.9,0.1</td>
<td>0.8,0.1</td>
</tr>
<tr>
<td>A3</td>
<td>0.8,0.3</td>
<td>0.9,0.1</td>
<td>0.8,0.1</td>
</tr>
<tr>
<td>A4</td>
<td>0.6,0.4</td>
<td>0.7,0.3</td>
<td>0.9,0.1</td>
</tr>
</tbody>
</table>

**Table 18. Assessment matrix $R'_3$**

<table>
<thead>
<tr>
<th></th>
<th>Early Stage</th>
<th>Middle Stage</th>
<th>Late Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C11</td>
<td>C12</td>
<td>C13</td>
</tr>
<tr>
<td>A1</td>
<td>0.8,0.2</td>
<td>0.6,0.4</td>
<td>0.9,0.1</td>
</tr>
<tr>
<td>A2</td>
<td>0.8,0.2</td>
<td>0.7,0.3</td>
<td>0.8,0.2</td>
</tr>
<tr>
<td>A3</td>
<td>0.8,0.2</td>
<td>0.9,0.2</td>
<td>0.8,0.3</td>
</tr>
<tr>
<td>A4</td>
<td>0.7,0.3</td>
<td>0.8,0.3</td>
<td>0.8,0.2</td>
</tr>
</tbody>
</table>

Step 2. The consistency measures between the three matrices $R'_1, R'_2, R'_3$ are calculated by Formula (2):
\[
C(R'_1, R'_2) = 0.5975, \quad C(R'_1, R'_3) = 0.5555, \quad C(R'_2, R'_3) = 0.5625
\]

Formula (3) is used to determine the overall consistency level of each group of decision-makers as follows:
\[
CC1=1.1531, \quad CC2=1.1601, \quad CC3=1.1180
\]

Calculate the weight of each group of decision-makers by using Formula (4): $w'_1 = 0.336059$, $w'_2 = 0.338096$, $w'_3 = 0.325845$.

Step 3. The group decision matrix $M'_i$ is formed by combining the weight of each group of decision-makers with their respective decision matrix through Formula (5), as shown in Table 19:

**Table 19. Group decision matrix $M'_i$**

<table>
<thead>
<tr>
<th></th>
<th>Early Stage</th>
<th>Middle Stage</th>
<th>Late Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C11</td>
<td>C12</td>
<td>C13</td>
</tr>
<tr>
<td>A1</td>
<td>0.8,0.2</td>
<td>0.8,0.3</td>
<td>0.8,0.4</td>
</tr>
<tr>
<td>A2</td>
<td>0.8,0.2</td>
<td>0.8,0.2</td>
<td>0.8,0.3</td>
</tr>
<tr>
<td>A3</td>
<td>0.8,0.2</td>
<td>0.9,0.2</td>
<td>0.8,0.3</td>
</tr>
<tr>
<td>A4</td>
<td>0.8,0.3</td>
<td>0.8,0.3</td>
<td>0.8,0.3</td>
</tr>
</tbody>
</table>

Step 4. The consistency measures of each group decision matrix and group decision matrix $M'_i$ are determined by Formula (6): $\theta_{(i)}' = 0.655828, \quad \theta_{(2i)}' = 0.662636, \quad \theta_{(3i)}' = 0.651796$.

Step 5. We set the consistency threshold to $\xi = 0.66$, then $\theta_{(i)}' < \xi$. The consistency level of the
third group of decision-makers is low, so they enter Step 6.

Step 6. \( \theta_{1(1)} > \theta_{2(1)} > \theta_{3(1)} \), the consistency level of the second group of decision-makers is the highest, so we use the second decision matrix as a reference to adjust the evaluation information in the third matrix that does not meet the consistency requirements. The adjusted evaluation information is shown in Table 20:

<table>
<thead>
<tr>
<th>Late Stage</th>
<th>Middle Stage</th>
<th>Early Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>C21</td>
<td>C22</td>
<td>C23</td>
</tr>
<tr>
<td>C24</td>
<td>C25</td>
<td>C26</td>
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<tr>
<td>C27</td>
<td>C28</td>
<td>C29</td>
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<tr>
<td>C30</td>
<td>C31</td>
<td>C32</td>
</tr>
<tr>
<td>C33</td>
<td>C34</td>
<td>C35</td>
</tr>
</tbody>
</table>

calculated by

Then, we return to Step 2, and the weights of each group of recalculated decision-makers are \( w_1' = 0.332629 \), \( w_2' = 0.338022 \), and \( w_3' = 0.32935 \). The new group decision matrix \( M_2' \) obtained according to the new weight and evaluation matrix is shown in Table 21.

<table>
<thead>
<tr>
<th>Late Stage</th>
<th>Middle Stage</th>
<th>Early Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>C21</td>
<td>C22</td>
<td>C23</td>
</tr>
<tr>
<td>C24</td>
<td>C25</td>
<td>C26</td>
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<tr>
<td>C27</td>
<td>C28</td>
<td>C29</td>
</tr>
<tr>
<td>C30</td>
<td>C31</td>
<td>C32</td>
</tr>
<tr>
<td>C33</td>
<td>C34</td>
<td>C35</td>
</tr>
</tbody>
</table>

calculated by

Then, calculate the consistency measure between each group decision matrix and group decision matrix \( M_2 \) through Formula (6): \( \theta_{1(2)}' = 0.656106 \), \( \theta_{2(2)}' = 0.671815 \), \( \theta_{3(2)}' = 0.668172 \), \( \theta_{4(2)}' < \xi \); therefore, the matrix \( R_{1(2)}' \) with the highest consistency level is still used as a reference to adjust the information in the matrix \( R_{1(2)}' \) to obtain a new evaluation matrix \( R_{1(2)}'' \), as shown in Table 22:

<table>
<thead>
<tr>
<th>Late Stage</th>
<th>Middle Stage</th>
<th>Early Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>C21</td>
<td>C22</td>
<td>C23</td>
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<tr>
<td>C24</td>
<td>C25</td>
<td>C26</td>
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<td>C27</td>
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<td>C29</td>
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<tr>
<td>C30</td>
<td>C31</td>
<td>C32</td>
</tr>
<tr>
<td>C33</td>
<td>C34</td>
<td>C35</td>
</tr>
</tbody>
</table>

Then, we return to Step 2, and the weights of each group of recalculated decision-makers are \( w_1'' = 0.336296 \), \( w_2'' = 0.335074 \), and \( w_3'' = 0.32863 \). The new group decision matrix \( M_3' \) obtained according to the new weight and evaluation matrix is shown in Table 23.

<table>
<thead>
<tr>
<th>Late Stage</th>
<th>Middle Stage</th>
<th>Early Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>C21</td>
<td>C22</td>
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<td>C24</td>
<td>C25</td>
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<td>C27</td>
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<td>C29</td>
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<tr>
<td>C30</td>
<td>C31</td>
<td>C32</td>
</tr>
<tr>
<td>C33</td>
<td>C34</td>
<td>C35</td>
</tr>
</tbody>
</table>
Then, calculate the consistency measures of each decision matrix and group decision matrix \( M_i \) through Formula (6): 
\[
\theta_{3(3)} = 0.674374, \quad \theta_{2(3)} = 0.676618, \quad \theta_{4(3)} = 0.67037
\]
and satisfies \( \theta_i > \xi \). At this time, the three groups of decision information matrices meet the consistency level requirements, so we think \( M_i \) the final group decision matrix.

References


[40] Chen K, Luo YD. Generalized orthopair linguistic muirhead mean operators and their application in multi-criteria


[47] Liu PD, Teng F. Some interval neutrosophic hesitant uncertain linguistic Bonferroni mean aggregation operators for multiple attribute decision-making. *Int J Uncertainty Quan.* 2017; 7(6).


