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Fuzzy-Based Parameter Uncertainty in 1-D Consolidation in Clay

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Abstract

Uncertainties in consolidation parameters are generally studied using probabilities. Due to lack of data and knowledge it is proposed to consider consolidation parameters within a fuzzy sets framework, as composed of series of intervals which allow taking into account uncertainty. One of the noticeable advantages of such an approach is its ability to deal easily with uncertainty in a context of lack of data. A fuzzy finite difference scheme is implemented and results compared with available data and with results from methods using probabilistic parameter distribution approaches for 1-D consolidation in clay.

Keywords: Coefficient of consolidation; Parameter uncertainty; Intervals; Fuzzy sets; Degree of uncertainty.

1. INTRODUCTION

Despite its widespread use, the one-dimensional consolidation theory in saturated clays considers in general soil parameters as deterministic entities with no variation or uncertainties. The coefficient of consolidation is an important parameter in the process, and it is affected by uncertainties which can be epistemic or aleatory. Epistemic uncertainty is related to lack of data or knowledge, while aleatory uncertainty is related to natural randomness (Baecher & Christian, 2003). Knowledge related uncertainty is also present in problems where engineering judgement is prevalent (Boumezerane et al., 2011; Jelusic & Zlender, 2014). In this study epistemic uncertainty is taken into consideration. It originates from estimating the parameter in laboratory or from field measurements (Vlcek et al., 2016). Shuklal et al. (2009) give an extended review on different techniques used for determining coefficient of consolidation c_v .

Studies on consolidation which involved uncertainty are usually based on probabilistic approaches, where data need to be abundant. Several research works were conducted using probabilistic approaches, among them the one from Chang (1985) who performed a study on parameter uncertainty in 1D consolidation in clay by considering the variability of c_v. In their research Abbasi et al. (2007) derived a one-dimensional non-linear partial differential equation for prediction of consolidation characteristics of soft clays considering variable values for c_v. They used a finite difference approach for the solution of a non-linear equation of consolidation.

On another aspect, Bong et al. (2018) considered spatial variability of natural soil deposits to obtain realistic probabilistically based estimates of the time for soil consolidation. They proposed two approaches for probabilistic analysis of consolidation considering the spatial variability of coefficient of consolidation.

The difficulty in using probabilistic approaches locates essentially in the need for large data to achieve accurate analyses. According to Baecher & Christian (2015) geotechnical engineer is facing problems with few data available, he or she faces extremely limited numbers of observations, measurements of differing types and quality, a blend of qualitative and quantitative information, and a need to make sequential decisions as data arrive.

The main objective of this research is to study the influence of parameter uncertainty in 1D consolidation. A fuzzy-based approach is developed to account for uncertainty in the consolidation coefficient c_v and its influence on evolution of excess pore pressure. The fuzzy-based approach is useful when dealing with vagueness and lack of data. The constructed fuzzy sets for c_v are taken as a superposition of intervals (α-level cuts).

2. THEORY AND METHODS

2.1 CONSOLIDATION IN SOIL

The change in volume $\frac{\Delta V}{V}$ is expressed via m_v, the coefficient of compressibility, as following

$$\frac{\Delta V}{V} = m_v \Delta\sigma' \qquad (1)$$

where $\Delta\sigma'$ is the increment of effective stress.

For the vertical direction

$$\frac{\Delta L}{L} = m_v \Delta\sigma' \qquad (2)$$

and m_v is given by

$$m_v = \frac{1}{E'}\left(1 - \frac{2\nu'^2}{1-\nu'}\right) \qquad (3)$$

E' and ν' are respectively the Young's modulus and Poisson's ratio.

The volume change in the soil skeleton is given by $\Delta V = m_v V \Delta\sigma'$.

The volume change of the pore water is given in terms of porosity n by $\Delta V = m_w n V \Delta u$. Water is practically incompressible, its compressibility m_w is negligible ($m_w = 0.5 \times 10^{-9} \frac{m^2}{N}$), u being the excess pore pressure.

The changes in volume must be equal since no water escapes

$$\Delta\sigma' = \frac{m_w}{m_v} \Delta u \qquad (4)$$

Equation (4) gives the relation between change in effective stress and change in excess pore pressure within the soil via a ratio between water compressibility m_w and soil compressibility m_v.

Consolidation in a uniform clay deposit can be simulated using the equation of dissipation of excess pore pressure (5), where values of the coefficient of consolidation c_v are considered constant.

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \qquad (5)$$

$$c_v = \frac{k}{m_v \gamma_w} \qquad (6)$$

where

k : Permeability of soil; m_v: Coefficient of volume compressibility; γ_w : Unit weight of water

The coefficient of volume compressibility can also be expressed in terms of void ratio

$$m_v = \frac{a_v}{1 + e_0}$$

where a_v: coefficient of compressibility, $a_v = \frac{\partial e}{\partial \Delta \sigma'}$, ∂e : Change in void ratio; $\partial \Delta \sigma'$: Change in effective stress.

The excess pore pressure u during consolidation is a function of soil properties, time, and elevation within the soil layer (z varying from 0 to H). The rate at which consolidation occurs clearly depends upon:

(a) The speed at which water can flow out of the soil, governed by the permeability k

(b) The volume of water that has to flow out of the soil, governed by the compressibility of the soil, and thus the parameter m_v

In this study uncertainty in c_v will be taken in terms of fuzzy sets, composed of a superposition of intervals. How does uncertainty propagate and influence excess pore pressure during the process of consolidation, is the main question of the problem.

2.2 UNCERTAINTY IN THE COEFFICIENT OF CONSOLIDATION c_v

According to Baecher & Christian, 2003 uncertainties in geotechnics can be classified into two categories, “epistemic” related to lack of data or knowledge and “aleatory” related to natural randomness. The consolidation parameter c_v is subjected to epistemic uncertainties which can influence the consolidation process in the soil. Different graphical methods are used for determining c_v (Taylor method, Casagrande method…), and the values of c_v are sometimes computed using formula (6), with a directly measured coefficient of permeability k and a coefficient of volume compressibility m_v obtained from an oedometer curve. These c_v

values have been found larger than the ones obtained by graphical methods (Shuklal et al., 2009). The value of c_v can increase during consolidation when m_v decreases at a rate higher than coefficient of permeability k, and decrease when k decreases at a rate higher than m_v.

In the considered formula of c_v (6) the main sources of uncertainty come from evaluation of permeability k and coefficient of volume compressibility m_v. In situ and laboratory measures are used to estimate permeability k in soils. The coefficient of volume compressibility m_v can be evaluated using $m_v = \frac{1}{E'}\left(1 - \frac{2\nu'^2}{1-\nu'}\right)$ in which uncertainty is induced principally from estimating E', the soil's effective modulus of elasticity.

Uncertainties are inherent to the testing method, and they propagate through different expressions used for the parameters of consolidation.

Among suitable techniques which can handle the perturbations fuzzy-based approaches gained widespread use in engineering. Fuzzy sets are often used to consider uncertainties which arise during experiments and propagate through measuring procedures and from the use of different devices.

From a mathematical point of view a fuzzy number can be decomposed into number of intervals or α-cuts. In the following section will be exposed the interval-based approach as a basis for constructing fuzzy numbers.

2.3 Reminder on intervals and fuzzy sets

Interval analysis was introduced by Moore (Moore, 1966) and is considered as a mathematical discipline that deals with quantities expressed as intervals which are common in engineering problems. Interval-valued approaches gained significant use in engineering especially when information is uncertain. It is a convenient tool to deal with uncertainty due to lack of data. As probabilistic approaches require large amount of data, when in geotechnics information is scarce and only small data is available in general, the use of interval analysis could be of interest (Moore et al., 2009). In practice, it may be difficult to get a large number of experimental data, consequently the need for an alternative method to handle uncertainty becomes essential. In this study, the consolidation parameter c_v will, first, be considered as interval-based. A superposition of intervals (α-level cut) leads to the construction of a fuzzy set for c_v.

Studies were conducted in different fields, such as thermal conductivity to account for uncertainty using interval approaches and fuzzy sets (Wang & Qiu, 2014). Wang & Qiu (2014) proposed a new numerical technique named as fuzzy finite difference method to solve the heat conduction problems with fuzzy uncertainties in both the physical parameters and initial/boundary conditions. The α-level cut method is used to study the problem in terms of interval equations. Kermani & Saburi (2007) presented a numerical method for solving "fuzzy partial differential equation" (FPDE) with some examples.

Using $\tilde{u}$ for fuzzy excess pore pressure, equation (5) can be written in fuzzy terms as following;

$$\frac{\partial \tilde{u}}{\partial t} = \widetilde{c_v} \frac{\partial^2 \tilde{u}}{\partial z^2} \qquad (7)$$

$\widetilde{c_v}$ is the fuzzy coefficient of consolidation. $\tilde{u}$ is the fuzzy excess pore pressure.

For the sake of representation the fuzzy numbers are written in terms of intervals at a given α-level cut.

$\tilde{u}$ is replaced by u_α^I . We write u_α^I for a given interval I of α-level cut. $\underline{u}(\alpha), \overline{u}(\alpha)$ are the lower and upper values of the interval at α-level cut. And $\underline{c_v}(\alpha), \overline{c_v}(\alpha)$ the lower and upper values of $\widetilde{c_v}$ at α-level cut.

A fuzzy set is a set of ordered pairs, $[x, \mu(x)]$, where a member x belongs to the set in a certain degree, termed membership grade $\mu(x)$. These ordered pairs collectively define a membership function that specifies a membership grade for each member (Zadeh, 1965). The triangular membership function, shown in Figure 1 is the most widely used membership function for representing uncertain soil parameters in geotechnical practice (Gong et al., 2014).

A triangular fuzzy number (Fig.1) is denoted by $\mathrm{u} = \langle \mathrm{a}, \mathrm{b}, \mathrm{c} \rangle$, where $\mathrm{a} \leq \mathrm{b} \leq \mathrm{c}$, has α-cuts (Klir & Yuan, 1995):

$$[\mathrm{u}]_\alpha = [\mathrm{a} + \alpha(\mathrm{b} - \mathrm{a}), \mathrm{c} - \alpha(\mathrm{c} - \mathrm{b})], \alpha \in [0,1]$$

And membership function $\mu(x)$

$$\mu(x) = \begin{cases} \frac{x-a}{b-a} & if\ a \leq x \leq b \\ \frac{c-x}{c-b} & if\ b \leq x \leq c \\ 0 & otherwise \end{cases} \qquad (8)$$

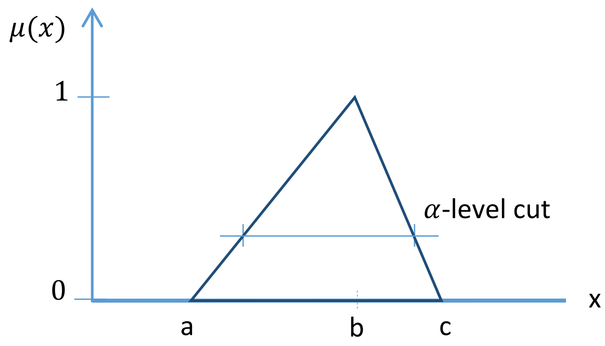

Figure 1 Fuzzy triangular number

Arithmetic operations on fuzzy numbers are based on interval arithmetic at α-level cut (Hanss, 2005).

Given intervals $[a_1, b_1]$, $[a_2, b_2]$

$[a_1, b_1] + [a_2, b_2] = [a_1 + a_2, b_1 + b_2]$ $M = \{a_1a_2, a_1b_2, b_1a_2, b_1b_2\}$

$[a_1, b_1] - [a_2, b_2] = [a_1 - b_2, b_1 - a_2]$

$[a_1, b_1] \times [a_2, b_2] = [\min(M), \max(M)]$

2.4 VALIDATION OF THE MATHEMATICAL APPROACH

Pirzada & Vakaskar (2017) presented an analytical solution for a fuzzy partial differential equation with fuzzy boundary and initial conditions. The solution of fuzzy heat equation is proposed using Seikkala differentiability of a fuzzy-valued function (Seikkala, 1987). The similarity of consolidation equation with heat equation allows using the same principles from Pirzada & Vakasar (2017) to work on a numerical solution using fuzzy sets.

The original fuzzy differential equation is given by

$$\frac{\partial \tilde{U}}{\partial t} = \tilde{P} \otimes \frac{\partial^2 \tilde{U}}{\partial x^2} \quad (9)$$

Where $\tilde{U}(x,t)$ is the fuzzy temperature, $\tilde{P}$ is the fuzzy thermal diffusivity and $\otimes$ is a multiplication operator between two fuzzy numbers.

By analogy one can write in terms of excess pore pressure $\tilde{u}$ the fuzzy differential equation of consolidation as

$$\frac{\partial \tilde{u}}{\partial t} = \widetilde{c_v} \otimes \frac{\partial^2 \tilde{u}}{\partial z^2} \quad (10)$$

Let's consider a certain α-level cut, one can write;

$$\tilde{u}_\alpha = \left[\underline{u}(\alpha), \overline{u}(\alpha)\right] \quad (11)$$

And

$$\frac{\partial \tilde{u}_\alpha}{\partial t} = \left[\frac{\partial \underline{u}(\alpha)}{\partial t}, \ \frac{\partial \overline{u}(\alpha)}{\partial t}\right] \quad (12)$$

$$\frac{\partial^2 \tilde{u}_\alpha}{\partial z^2} = \left[\frac{\partial^2 \underline{u}(\alpha)}{\partial z^2}, \frac{\partial^2 \overline{u}(\alpha)}{\partial z^2}\right] \quad (13)$$

$$\widetilde{c_v}_\alpha = \left[\underline{c_v}(\alpha), \overline{c_v}(\alpha)\right]$$

(10), (11) (12) and (13) are expressions of $\tilde{u}$ and derivatives at α-level cut.

Using differentiability of $\tilde{u}$ and fuzzy arithmetic, the fuzzy equation can be written as the system of parametric form of consolidation equation

$$\frac{\partial \underline{u}(\alpha)}{\partial t} = min\left\{\underline{c_v}(\alpha)\frac{\partial^2 \underline{u}(\alpha)}{\partial z^2}, \ \underline{c_v}(\alpha)\frac{\partial^2 \overline{u}(\alpha)}{\partial z^2}, \ \overline{c_v}(\alpha)\frac{\partial^2 \underline{u}(\alpha)}{\partial z^2}, \ \overline{c_v}(\alpha)\frac{\partial^2 \overline{u}(\alpha)}{\partial z^2}\right\} \quad (14)$$

$$\frac{\partial \overline{u}(\alpha)}{\partial t} = max\left\{\underline{c_v}(\alpha)\frac{\partial^2 \underline{u}(\alpha)}{\partial z^2}, \ \underline{c_v}(\alpha)\frac{\partial^2 \overline{u}(\alpha)}{\partial z^2}, \ \overline{c_v}(\alpha)\frac{\partial^2 \underline{u}(\alpha)}{\partial z^2}, \ \overline{c_v}(\alpha)\frac{\partial^2 \overline{u}(\alpha)}{\partial z^2}\right\} \quad (15)$$

In the solution proposed by Pirzada & Vakaskar (2017) equations (14) and (15) can be simplified depending on the signs of $\frac{\partial^2 \underline{u}(\alpha)}{\partial z^2}, \frac{\partial^2 \overline{u}(\alpha)}{\partial z^2}$

- Case 1: $\frac{\partial^2 \underline{u}(\alpha)}{\partial z^2}, \frac{\partial^2 \overline{u}(\alpha)}{\partial z^2} > 0$

  In this case the system of equations (14) and (15) will be studied as

$$\frac{\partial \underline{u}(\alpha)}{\partial t} = \underline{c_v}(\alpha) \frac{\partial^2 \underline{u}(\alpha)}{\partial z^2} \tag{16}$$

$$\frac{\partial \overline{u}(\alpha)}{\partial t} = \overline{c_v}(\alpha) \frac{\partial^2 \overline{u}(\alpha)}{\partial z^2} \tag{17}$$

- Case 2: : $\frac{\partial^2 \underline{u}(\alpha)}{\partial z^2} < 0, \frac{\partial^2 \overline{u}(\alpha)}{\partial z^2} > 0$

  The system of equations will simplify to

$$\frac{\partial \underline{u}(\alpha)}{\partial t} = \overline{c_v}(\alpha) \frac{\partial^2 \underline{u}(\alpha)}{\partial z^2} \tag{18}$$

$$\frac{\partial \overline{u}(\alpha)}{\partial t} = \underline{c_v}(\alpha) \frac{\partial^2 \overline{u}(\alpha)}{\partial z^2} \tag{19}$$

- Case 3: $\frac{\partial^2 \underline{u}(\alpha)}{\partial z^2} < 0, \frac{\partial^2 \overline{u}(\alpha)}{\partial z^2} < 0$ The system reduces to equations (18) and (19)

Based on the framework of the analytical solutions a numerical approach using finite difference method is applied to solve the consolidation equation in terms of intervals and by extension in terms of fuzzy numbers. It allows considering different boundary conditions and variations of the parameters.

The initial and boundary conditions are:

- At initial time $t = 0$ the distribution of excess pore pressure is uniform along the height (z) and is given by $u(0, z) = \Delta q$

  Δq: Increment of pressure
- At depth $z = 0$ and $z = H$ at $t \neq 0$ the excess pore pressure is zero (drained boundaries) $u(t, 0) = u(t, H) = 0$

$$\frac{\partial \tilde{u}}{\partial t} = \frac{\partial[\underline{u}(\alpha), \overline{u}(\alpha)]}{\partial t}$$

At a given α-level cut the term u_α^I can be used for upper or lower value of u in the interval I.

$$\frac{\partial u_\alpha^I}{\partial t} \approx \frac{u_\alpha^{I.i+1} - u_\alpha^{I.i}}{\Delta t} \qquad (20)$$

$$\frac{\partial^2 \tilde{u}}{\partial z^2} = \frac{\partial^2 u_\alpha^I}{\partial z^2} \approx \frac{u_{\alpha,j+1}^I - 2u_{\alpha,j}^I + u_{\alpha,j-1}^I}{(\Delta z)^2} \qquad (21)$$

The forward scheme of finite difference approach using interval-based parameters can be written for α-level cuts used to characterize a fuzzy number. Equation (22) is written for one α-level cut with an interval denoted by I. The coefficient of consolidation $\tilde{c_v}$ is taken as a constant fuzzy number.

$$u_{\alpha,j}^{I,i+1} = u_{\alpha,j}^{I,i} + {c_v}_{\alpha}^{I} \frac{\Delta t}{(\Delta z)^2} [u_{\alpha,j+1}^{I,i} - 2u_{\alpha,j}^{I,i} + u_{\alpha,j-1}^{I,i}] \quad (22)$$

where

$${c_v}_{\alpha}^{I} = \frac{1}{\gamma_w} (\frac{k}{m_v})_{\alpha}^{I} \quad (23)$$

γ_w is considered as crisp number during analysis

In situ and laboratory measures are used to evaluate permeability k in soils. The coefficient of volume compressibility m_v can be evaluated using

$m_v = \frac{1}{E'}\left(1 - \frac{2\nu'^2}{1-\nu'}\right)$; E', ν' elastic parameters.

3 EXAMPLE FOR MODEL CALIBRATION

For calibration of the described model we use crisp values of c_v and perform calculations in comparison to existing theoretical solutions of 1-D consolidation. Figure 2 shows evolution of excess pore pressure versus height for different time factors T_v, analytical solutions from Terzaghi and numerical solutions from Stickle & Pastor (2018) are presented. In figure 3 is plotted the excess pore pressure with depth for different time factors T_v using the finite difference scheme of the present study. Figure 4 shows the evolution of the maximum relative pore pressure $\frac{u}{u_0}$ with time.

The model was run using the following deterministic soil properties:

Elastic modulus $E' = 1000 \frac{kN}{m^3}$, $\nu' = 0$ (Oedometric condition) and a uniformly distributed initial pore pressure $u_0 = 1 \frac{kN}{m^2}$ with a permeability $k = 0.001 \frac{m}{day}$ (Stickle & Pastor, 2018). At initial condition we consider a uniform distribution of the pore pressure along the depth $H = 4m$ equal to the applied load $q_0 = 1\ \frac{kN}{m^2}$. Drainage was considered open on top and bottom of the soil layer.

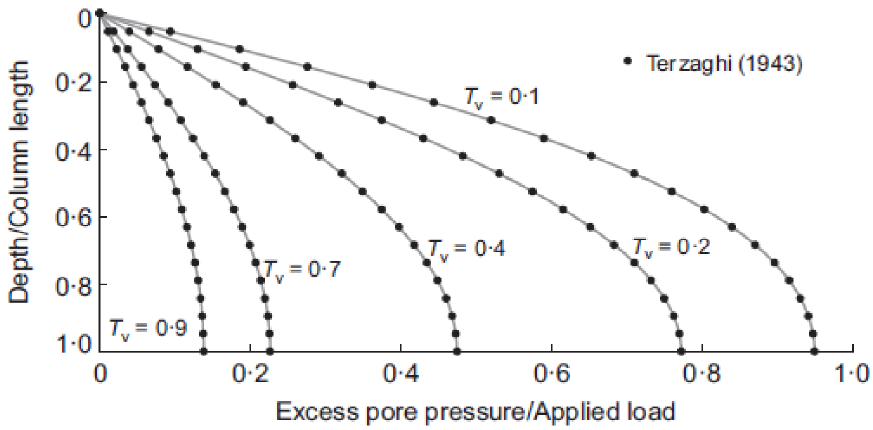

Figure 2 Solutions of Terzaghi equation, analytical and numerical (Stickle & Pastor, 2018)

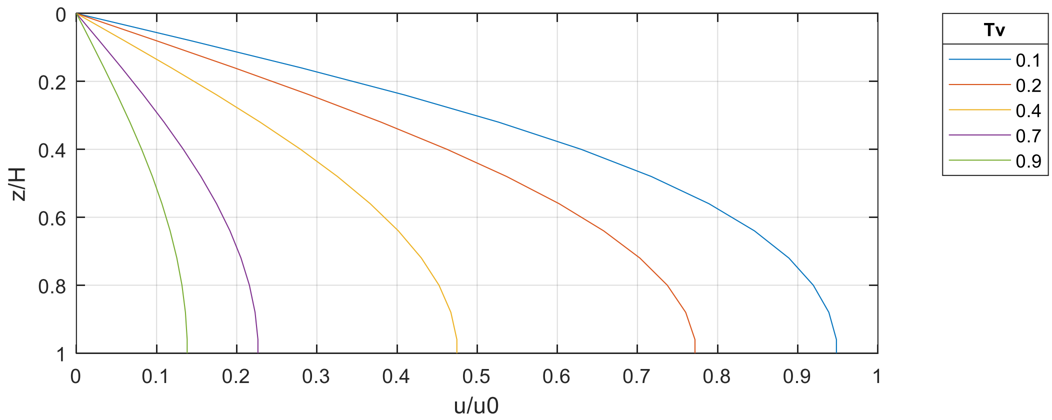

Figure 3 Relative excess pore pressure $\frac{u}{u_0}$ versus relative depth $\frac{z}{H}$ for different values of time factor T_v (present study)

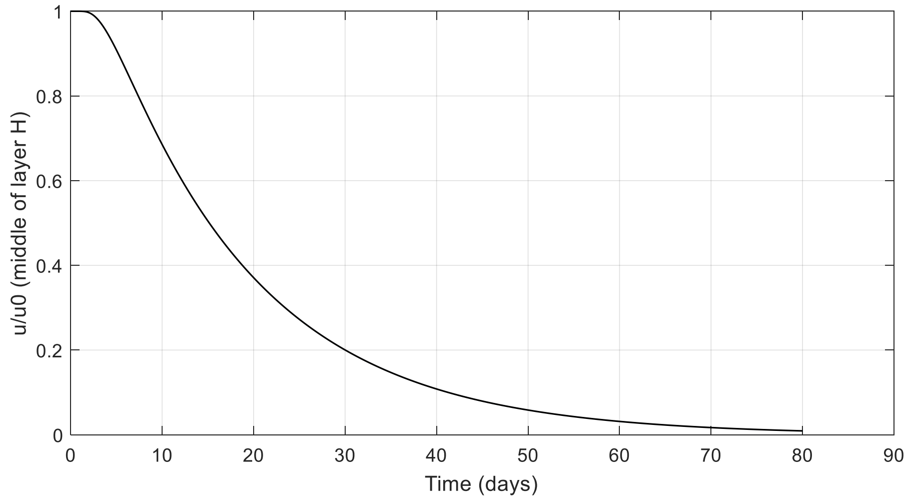

Figure 4 Evolution of maximum relative pore pressure $\frac{u}{u_0}$ with time (In the middle of the layer) - present study

4 ANALYSIS AND DISCUSSION OF RESULTS

In this section a comparison is made between the results of the proposed fuzzy-based approach with previous analysis based on probability tools. Chang (1985) analyzed uncertainty in one-dimensional consolidation using a probabilistic approach. The measured coefficient of consolidation, c_v can have a substantial degree of variation as shown in figure 5. The author studied the adequacy of a gamma distribution model for c_v which was tested using experimental data. The results were discussed in terms of confidence level with examples to demonstrate the variability of the solution of consolidation analysis.

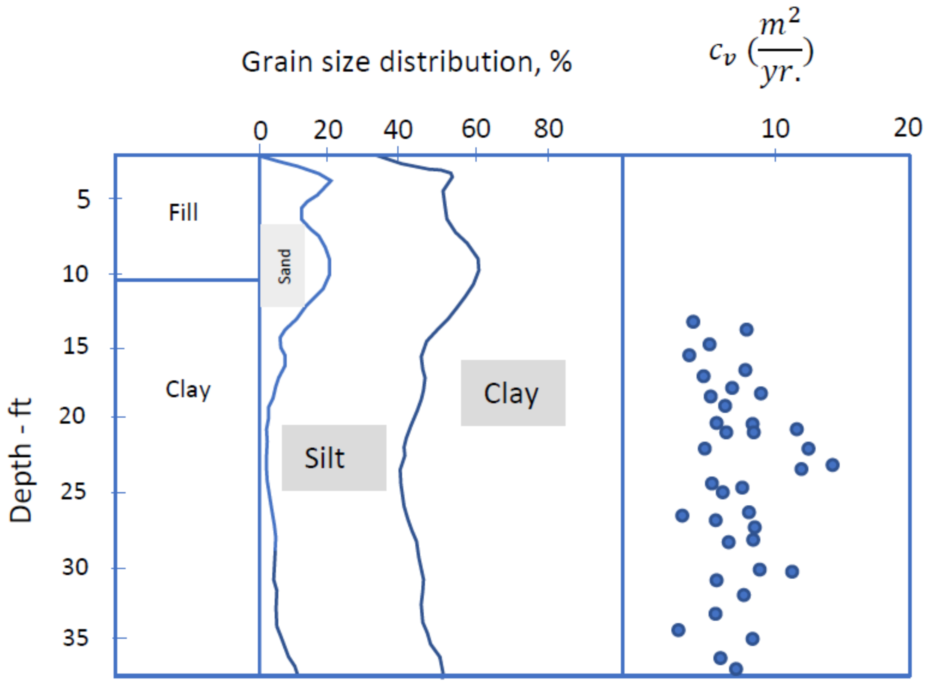

Figure 5 Variation of c_v values of Kawasaki clay (Chang, 1985)

The values of c_v were measured for soil samples from different locations in a clay layer at different depths between 3.66m and 10.97m below the ground surface. c_v can have a degree of variation even in a uniform clay layer. Its values range from 3.8 $\frac{m^2}{year}$ and 13.2 $\frac{m^2}{year}$, with a mean 7.59 $\frac{m^2}{year}$.

A degree of variability typical of many types of clay as mentioned by the authors. In their study c_v has no correlation with depth.

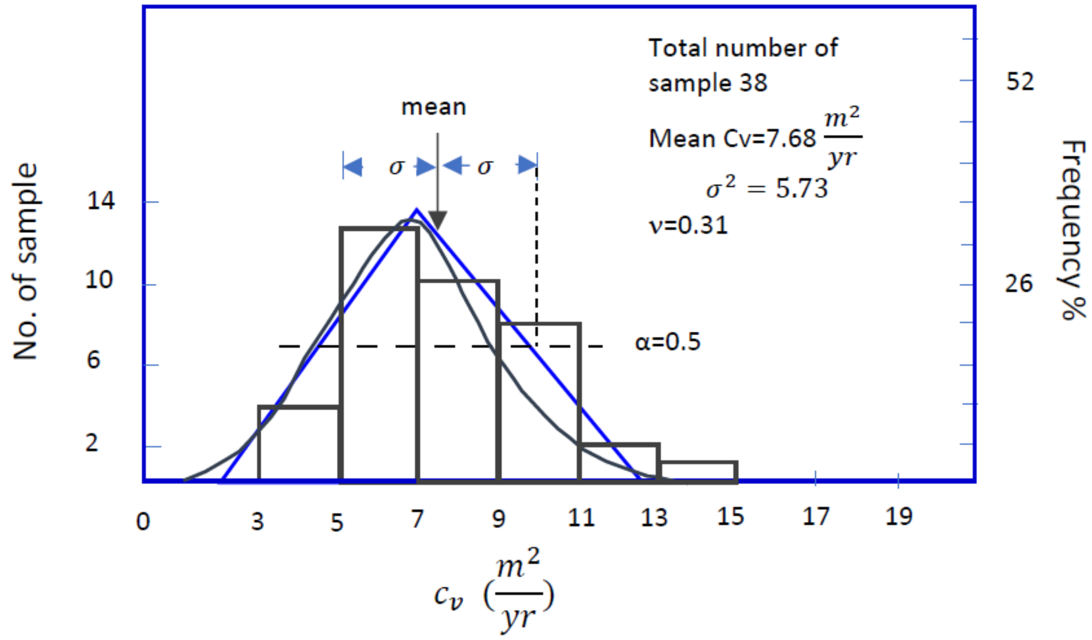

Figure 6 Approximation of histogram of values of c_v by a triangular fuzzy set $\tilde{c_v}$

(Kawasaki clay, Chang 1985)

Hanss & Willner (2000) showed that a normal probability distribution constructed from measured parameter data can be approximated by a Gaussian or a triangular fuzzy number. For sparse data one can use the approach proposed by Dridger et al. (2018) based on probability-possibility transformation. The distribution of c_v in figure 6 is not symmetric, and can be approximated using a non-symmetric triangular fuzzy number. Using the probability distribution (histogram) a triangular fuzzy number $\tilde{c_v}$ is proposed for the coefficient of consolidation in which the membership function is given using expression (8);

where $$a = 2,\ b = 7,\ c = 12.8\frac{m^2}{year}$$

In $\frac{m^2}{day}$ one gets $a = 5.48 \times 10^{-3}, b = 19.18 \times 10^{-3}, c = 35.06 \times 10^{-3} \frac{m^2}{day}$

Given an α-level cut of $\tilde{c_v}_\alpha = \left[\underline{c_v}(\alpha), \overline{c_v}(\alpha)\right]$, the degree of uncertainty of consolidation coefficient is defined according to Datta (2017) by:

$$\Delta c_v|_\alpha = \left[\frac{\overline{c_v}(\alpha) - \underline{c_v}(\alpha)}{\overline{c_v}(\alpha) + \underline{c_v}(\alpha)}\right] \quad (24)$$

Equation (24) is a measure of uncertainty given a fuzzy representation of the coefficient of consolidation $\tilde{c_v}$. Different measures of uncertainty were proposed when dealing with fuzzy parameters. Yager proposed fuzziness measure based on the difference between fuzzy set A and its complement, given interval support (El Hawy et al.,2015).

The degree of uncertainty of $\tilde{c_v}$ using expression (24) is represented in figure 7.

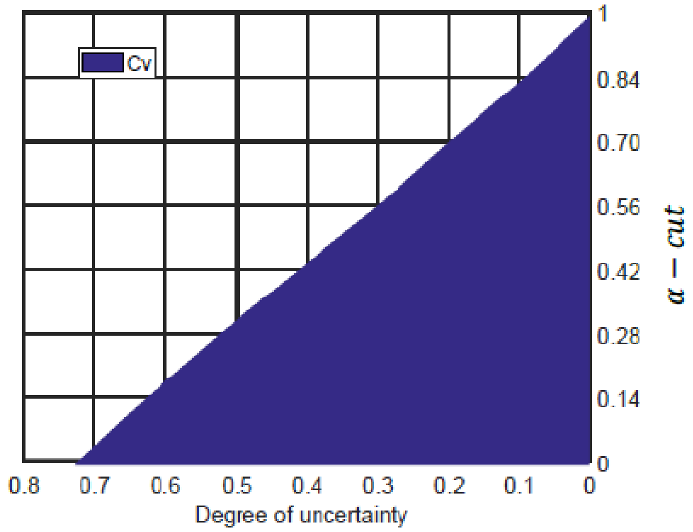

Figure 7. Degree of uncertainty in the fuzzy coefficient of consolidation $\tilde{c_v}$

Figure 8 shows results from Chang (1985) in terms of confidence band for maximum excess pore pressure ($\frac{u}{u_0}$) as a function of the mean of time factor. The confidence band is plotted for one standard deviation 1σ and for 1.5σ , the idea is to present the evolution with time of uncertainty in the excess pore pressure. It is clear from the curve that uncertainty goes increasing with time as the process of consolidation occurs. The band is becoming larger. Based on similar reasoning, in this fuzzy-based approach we used the central (peak) value of the triangular fuzzy number $\tilde{T}_v$ to plot curves of maximum excess pore pressure within the same conditions of consolidation (Figure 9). The expression of the fuzzy time factor for the double drained layer H is given by $\tilde{T}_v = \frac{\tilde{C}_v\, t}{\left(\frac{H}{2}\right)^2}$.

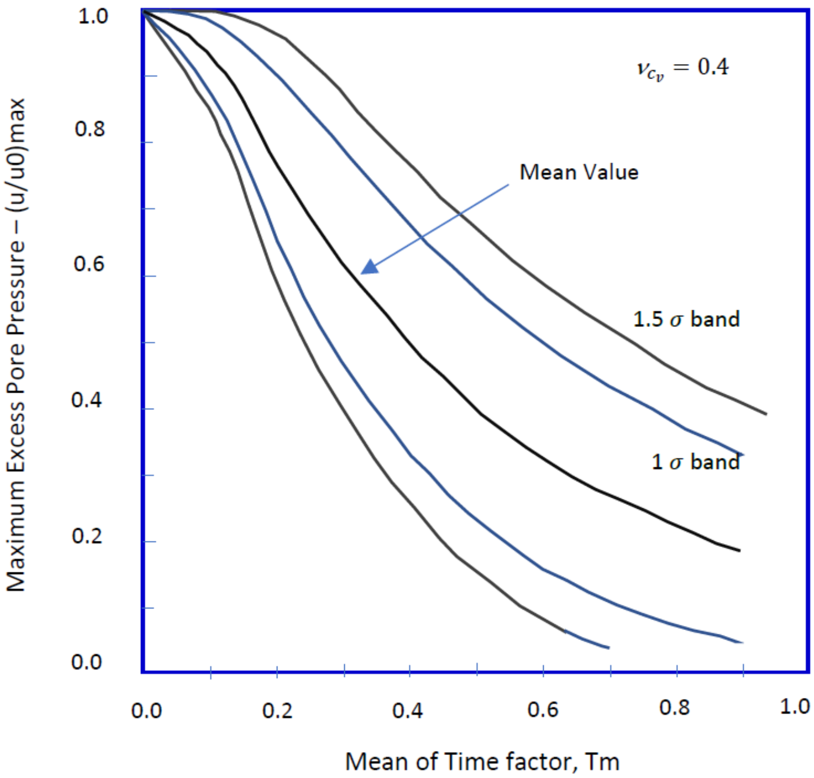

Figure 8. Confidence Band for Maximum Excess Pore Pressure $\frac{u}{u_0}$ $v_{c_v} = 0.4$ (Chang, 1985)

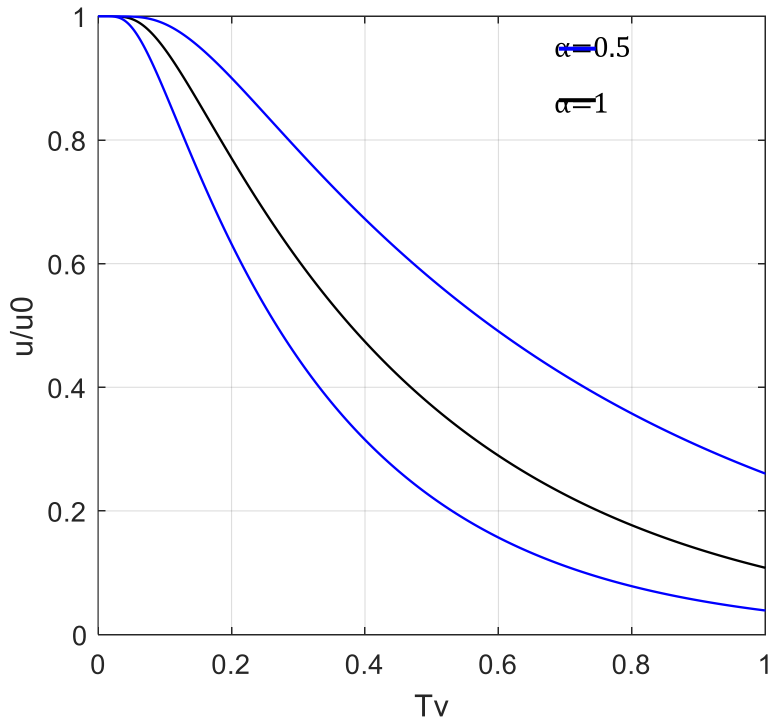

Figure 9. Max Excess pore pressure $\frac{u}{u_0}$ versus peak time factor Tv – Results at α- level cut = 0.5 and α = 1

The 1-σ band in the probabilistic approach of Chang (1985) corresponds approximately to an α-level cut of 0.5 in the obtained fuzzy distribution of excess pore pressure. The results of the approach using the approximated fuzzy set for c_v show the same behaviour of $\frac{u}{u_0}$ with the one obtained by Chang (1985) for 1-σ band (figures 8,9). Between figure 8 and 9 the trend is similar and the evolution of maximum excess pore pressure using fuzzy approximation follows identical progress as using 1-σ confidence band. It is noticeable that the fuzzy set used to estimate the coefficient of consolidation c_v from the given histogram (probability distribution) is an accurate approach to handle uncertainty. This approach can be used in case of scarce information as long as the fuzzy set covers most of available data.

Figures 10-13 show fuzzy sets of Max $\frac{u}{u_0}$ (excess pore pressure) at different time factors (peak) during consolidation.

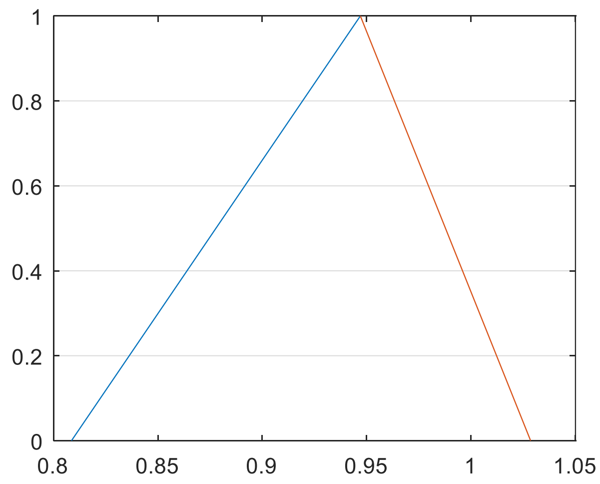

Figure 10 Fuzzy set of Max $\frac{u}{u_0}$ at $T_v = 0.1$

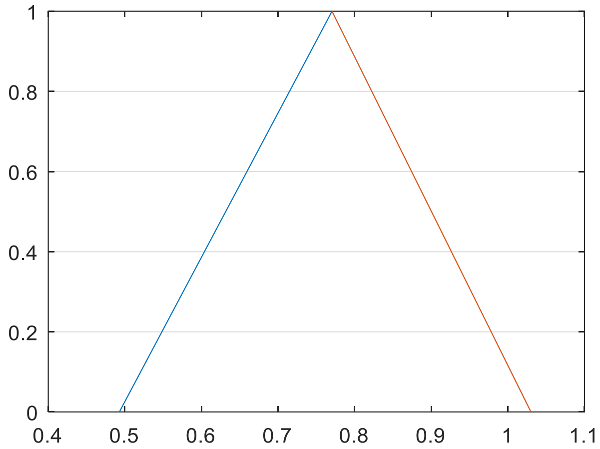

Figure 11 Fuzzy set of Max $\frac{u}{u_0}$ at $T_v = 0.2$

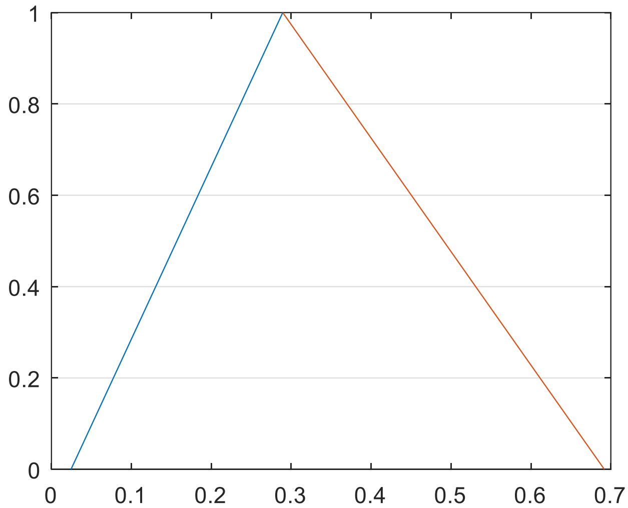

Figure 12 Fuzzy set of Max $\frac{u}{u_0}$ at $T_v = 0.6$

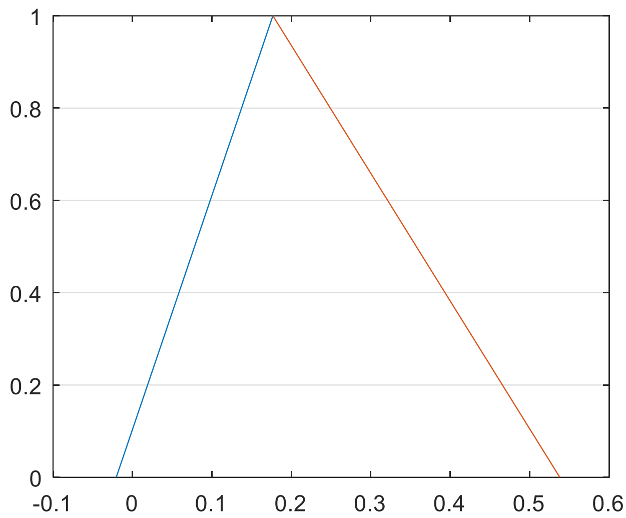

Figure 13 Fuzzy set of Max $\frac{u}{u_0}$ at $T_v = 0.8$

The shape of excess pore pressure fuzzy sets remained triangular with slight differences in terms of behaviour. Uncertainty propagates from the use of a triangular fuzzy representation of c_v, conserving the triangular shape in excess pore pressure also. The propagation of uncertainty during the process of consolidation can be calculated using expression (24). It is observable that the variability of Max $\frac{u}{u_0}$ which starts small, becomes larger at $T_v = 0.6$ and continues in spreading uncertainty when the process is close to $T_v = 1$.

In figure 14 are plotted the shapes and areas of uncertainty measures (degree of uncertainty) of Max $\frac{u}{u_0}$ at different time factors T_v using equation (24). Each coloured area of the figure represents the level of uncertainty at a given time of the consolidation process. The change can be noticed in the degree of uncertainty as long as time passes. In the beginning of the consolidation process ($T_v = 0.1$) the degree of uncertainty is small at different α-cuts, the variation becomes perceptible for $T_v = 0.6$ and $T_v = 0.8$. Mainly the shape of the degree of uncertainty of Max $\frac{u}{u_0}$ represented by coloured areas stays triangular at $T_v = 0.1$ and $T_v = 0.2$ but changes slightly to a curved form when $T_v = 0.8$ reproducing the uncertainty contained in the input parameter of consolidation.

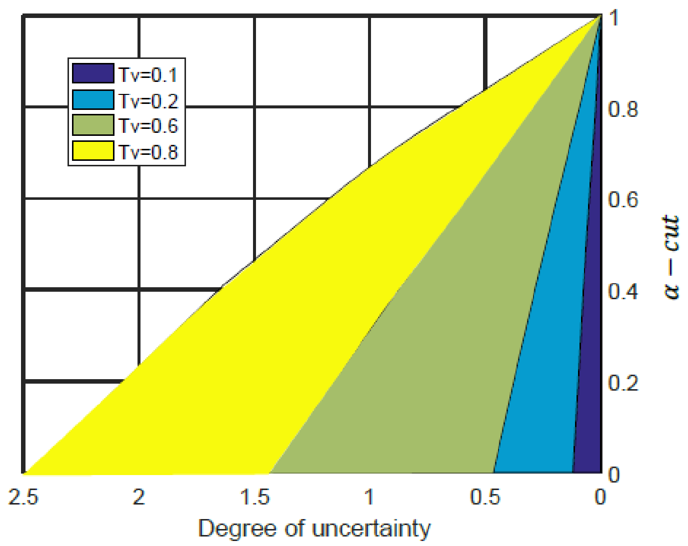

Figure 14. Evolution of uncertainty in Max $\frac{u}{u_0}$ at different time factors Tv

5 CONCLUSION

In this study an attempt is made to show the influence of parameter uncertainty in 1-D consolidation process using fuzzy sets. An approach is proposed by considering the coefficient of consolidation as a fuzzy number based on superposition of intervals (α – level cut). An Interval-based Finite Difference scheme is utilized to solve 1-D consolidation equation.

It was shown from different simulations that uncertainty can be well perceived when working with fuzzy sets. Available data from laboratory and other sources was utilized in this approach. The main advantage is in its ability to handle uncertainty easily and most importantly it can be employed when there is lack of data. The obtained results using the present approach are in good agreement with results from a probabilistic approach. In this study, information can be handled easily, fuzzy arithmetic is of great interest.

The degree of uncertainty can be measured using information available in the fuzzy set, either input or output. The propagation of uncertainty within the process of 1D consolidation was shown through measuring $\Delta c_v|_\alpha$ and its influence on the output fuzzy set of excess pore pressure. It helps mapping out uncertainty during consolidation.

References

Abbasi N, H Rahimi, AA Javadi & A Fakher (2007) Finite difference approach for consolidation with variable compressibility and permeability. *Computers and Geotechnics*. Volume 34, Issue 1, pp 41-52

Baecher GB & Christian JT (2003*) Reliability and Statistics in Geotechnical Engineering*. Wiley.

Baecher GB & JT Christian (2015) Bayes Factors and the Observational Method. *Geotechnical Safety and Risk V T. Schweckendiek et al. (Eds.)*

Bong T, AW Stuedlein (2018) Efficient methodology for probabilistic analysis of consolidation considering spatial variability. *Engineering Geology*, Volume 237, April 2018, pp 53-63.

Boumezerane D, Belkacemi S & Žlender B. (2011) Fuzzy-sets decision-support system for geotechnical site soundings. *Acta Geotechnica Slovenica*, *2011*(1), 51-63.

Chang CS (1985) Uncertainty in One-Dimensional Consolidation Analysis. *Journal of Geotechnical Engineering*, Vol.111, Issue 12 – December 1985.

Datta D (2017) An algorithm for solving fuzzy advection diffusion equation and its application to transport of radon from soil into buildings. *International Journal System Assurance Engineering and Management* 8, 2129–2136. https://doi.org/10.1007/s13198-014-0319-1

Dridger A, I Caylak, R Mahnken & E Penner (2018) A possibilistic finite element method for sparse data, Safety and Reliability, 38:1-2, 58-82.

El_Hawy Mohamed AH, HA Hassan, HA Hefny & KT Wassif (2015) An Improved Fuzzy Number Approximation using Shadowed Sets**,** *International Journal of Computer Applications* (0975 – 8887) Volume 118 – No.25

Gong W, L Wang, S Khoshnevisan, CH Juang , H. Huang & J. Zhang (2014) Robust Geotechnical Design of Earth Slopes Using Fuzzy Sets. *Journal of Geotechnical and Geoenvironmental Engineering,* 2014, DOI: 10.1061/ (ASCE)GT.1943-5606.0001196

Hanss M (2005) *Applied Fuzzy Arithmetic, An Introduction with Engineering Applications*. Springer Verlag

Hanss M, K Willner (2000) On Using Fuzzy Arithmetic To Solve Problems With Uncertain Model Parameters. *International Journal Of Project Management*, Volume 20, Issue 3.

Jelušič P & B Žlender (2014) An Adaptive Network Fuzzy Inference System Approach for Site investigation. *Geotechnical Testing Journal* 37, no. 3 (2014): 400-411. https://doi.org/10.1520/GTJ20120018

Kermani MA & F Saburi (2007) Numerical Method for Fuzzy Partial Differential Equations. *Applied Mathematical Sciences,* Vol. 1, no. 27, pp 1299 – 1309

Klir G & B Yuan (1995) *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. Prentice-Hall.

Moore RE. (1966) *Interval Analysis*. Prentice-Hall, Englewood Cliff, NJ.

Moore RE, RB Kearfott & MJ Cloud (2009*) Introduction to interval analysis*. Society for Industrial and Applied Mathematics SIAM, Philadelphia

Pirzada S & DJ Vakaskar (2017) Solution of Fuzzy Heat Equation Under Fuzzified Thermal Diffusivity. *P. Manchanda et al. (eds.), Industrial Mathematics and Complex Systems. Industrial and Applied Mathematics, DOI 10.1007/978-981-10-3758-0_19*

Seikkala S (1987) On the fuzzy initial value problem. *Fuzzy Sets and Sytems*, vol.24, no.3, pp.319-330.

Shuklal SK, N Sivakugan,& BM Das (2009) Methods for determination of the coefficient of consolidation and field observations of time rate of settlement — an overview. *International Journal of Geotechnical Engineering.* 3: (89-108) DOI 10.3328/IJGE.2009.03.01.89-108

Stickle MM & Pastor M. (2018). A practical analytical solution for one-dimensional consolidation. *Géotechnique* 68, No. 9, pp 786–793.

Vlcek J, R Bulko, L Bartuška, & J Kmec (2016) Soil Consolidation Parameters Derived from CPTu Probing. *Procedia Engineering* Volume 161, 2016, pp 180-184

Wang C. & ZP Qiu (2014) Fuzzy finite difference method for heat conduction analysis with uncertain parameters. *Acta Mechanica Sinica* 30(3), pp 383–390. DOI 10.1007/s10409-014-0036-7

Zadeh L (1965) Fuzzy Sets. *Information and Control*. Volume 8, Issue 3, pp 338-353.