Bayesian updating of uncertainties in the stability analysis of natural slopes in sensitive clays

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Abstract: Safety assessment of natural slopes in sensitive clays is subjected to uncertainty due to the natural variation of soil properties, measurement and modelling errors. In order to ensure acceptable safety levels, geotechnical design codes (e.g., Eurocode 7) commonly provide frameworks for a systematic treatment of uncertainties in the safety assessment of a slope. The treatment of uncertainties in the design codes is primarily focused on the parameters directly involved in the analysis of the mechanical stability of a slope (e.g., soil strength parameters). Additional valuable contributions to the safety assessments of slopes can be also provided by information that relates indirectly to the mechanical stability of a slope (e.g., past slope performance). However, there is often a lack of systematic integration of indirect information in the existing design codes. This paper examines the integration of indirect information based on the observed past slope performance in the safety assessment of a slope. The integration is facilitated through the Bayesian framework because it provides a basis to update uncertainties in the slope stability and safety assessment, such that they are consistent to the observed slope performance. The paper examines the effects of slope survival and failure events on uncertainties in the slope stability analysis.

Introduction

The stability of slopes in sensitive clays is a geotechnical problem of high importance due to the social and economic consequences commonly associated with the corresponding failures. A good appraisal of slope stability provides a basis for the evaluation of hazards and consequences imposed by a potential slope failure. An appraisal of slope stability relies on a wide range of information provided by in-situ and laboratory soil tests, load predictions, engineer's experience, site geology, surveying, past slope performance etc. In order to ensure acceptable safety levels, geotechnical design codes (e.g., Eurocode 7 (EU, 2004), ISO (ISO, 2010)) commonly provide a systematic framework for the treatment of different information in the evaluation of a slope stability. The focus of the design codes is mainly on the information directly involved in the analysis of mechanical stability of a slope (e.g., soil strength parameters, loads). Conversely, there is often a lack of systematic treatment of information contributing indirectly to the analysis of mechanical stability of slopes, as for example past slope performance.

Despite the large number of international design codes, relatively few provide the necessary framework to integrate the indirect information obtained by past slope performance in the safety assessment of a slope (e.g., ISO 13822 (ISO, 2010) and SIA 269/7 (e.g., Brühwiler et al., 2012). When it comes to the stability analysis of sensitive clays, significant efforts on the integration of information based on the past performance were conducted within the Natural hazards, infrastructure, floods and landslides (NIFS) project (e.g., Myrabø S., 2016). The NIFS project introduced the percentage improvement method (e.g., Oset et al., 2014) to account for indirect information in the safety assessment of a slope. The percentage improvement method adjusts the safety assessment of a slope based on the observed slope performance by considering that a stable natural slope at a given location has a factor of safety of at least one (Oset et al., 2014). The uncertainties are updated implicitly by recalibrating the safety assessment of a slope in terms of the factor of safety. The required safety levels are achieved by implementing a relative (i.e., percentage) improvement to the initially estimated factor of safety. The level of the relative improvement decreases proportionally with the level of agreement between the initially estimated and the observed slope performance in terms of the factor of safety.

In addition to the recalibration of the safety assessment, the information on the observed slope performance can be valuable for risk management, upgrading analysis, repair design and post-failure analysis of slopes. This is based on the consideration that the observed performance of a slope (e.g., survival of a certain load) can be characterized as an outcome of a full-scale test on the slope. In contrast to the percentage improvement method, where the uncertainties in the safety assessment are implicitly accounted for, this paper provides an explicit updating of uncertainties. The explicit updating of the uncertainties is facilitated through the implementation of the probabilistic Bayesian framework. The Bayesian framework is highly flexible in updating uncertainties in engineering systems based on information of the system performance (e.g., Straub & Papaioannou, 2014). The framework has a wide range of applications in geotechnical engineering. For example, the application of the Bayesian framework for updating of slope safety assessment based on observed performance was also considered in Gilbert et al. (1998); Luckman et al. (1987); Zhang et al. (2010). The Bayesian framework was applied to update uncertainties in the safety assessment and of deep excavations in Hsein Juang et al. (2012) and Papaioannou and Straub (2012). Depina et al. (2016) applied the Bayesian framework to integrate different sources of information in the classification of CPT data. The application of Bayesian updating to dike safety was examined in Schweckendiek et al. (2014).

This study employs the Bayesian framework to examine the effects of information provided by a survival and a failure event on uncertainties in the safety assessment of a slope. In comparison to the previous studies on slope safety assessment this study employs a relatively simple direct Bayesian updating approach, implemented in Schweckendiek et al. (2014). The advantages of the Bayesian framework for updating problems related to the safety assessment of slopes in sensitive clay are examined on numerical examples of a slope in drained and undrained conditions.

Problem definition

The stability of a slope is evaluated by a model comprised of a set of equations and boundary conditions that specify material properties, geometry, and loads. Due to the various sources of uncertainty, the parameters of a slope stability model are often subjected to uncertainties (e.g., spatially variable soil properties). Consider an n-dimensional vector of random parameters, $\mathbf{X}=[X_1,...,X_n]^T$, that corresponds to the uncertain parameters in a slope stability model. $\mathbf{X}$ is distributed according to a joint probability density function (pdf), $f(\mathbf{x})$, where $\mathbf{x}$ is a realization of $\mathbf{X}$ in the corresponding outcome space, $\Omega_{\mathbf{X}}$. A slope stability model often includes a series of assumptions and simplifications that introduce additional uncertainties in a slope stability analysis. The model uncertainty is commonly modeled by introducing one or more additional random variables to $\mathbf{X}$ (e.g., Straub & Papaioannou, 2014).

The aim of this study is to utilize observations of a slope performance to learn about the uncertain parameters in a slope stability analysis. The Bayesian framework can be applied in such problems, because it allows one to update a prior model of uncertainties, $f'(\mathbf{x})$, with observations to a posterior probability distribution, $f''(\mathbf{x})$, as follows:

$$f''(\mathbf{x}) = \frac{L(\mathbf{x})f'(\mathbf{x})}{\int_{\Omega x} L(\mathbf{x})f'(\mathbf{x})d\mathbf{x}} \quad (1)$$

where $L(\mathbf{x})$ α Pr(Observation | $\mathbf{X}=\mathbf{x}$) is the likelihood function, proportional to the probability of observing a given value of model parameters. The evaluation of the likelihood function is one of the central elements in the Bayesian framework. In the case of the Bayesian updating of engineering models, the likelihood function relies on a link between an observed performance and the corresponding model prediction. Let y denote the observed performance, while $h(\mathbf{X})$ represents the corresponding model prediction. Due to measurement or prediction errors, it is common to detect a certain deviation between the observation and the model prediction, $\varepsilon=y-h(\mathbf{X})$. The deviations are uncertain due to uncertain model predictions and distributed according to a pdf f_ε. The distribution of deviations provides a basis for the construction of the likelihood function as $L(\mathbf{x}) = f_\varepsilon\left[y-h(\mathbf{x})\right]$.

Depending on the type of an information provided by an observation, two classes of observations can be defined as follows (e.g., Straub & Papaioannou, 2014):

Class 1: Observation providing inequality information (e.g., slope survival corresponds to a factor of safety of larger than one). A model response can be formulated such that the observation is defined as an event $Z = [\mathbf{x} \in \Omega_{\mathbf{x}} : h(\mathbf{x}) < 0]$. The corresponding likelihood function is $L(\mathbf{x}) = \Pr(Z \mid \mathbf{X} = \mathbf{x}) = I[h(\mathbf{x}) < 0]$, where I is the indicator function that takes the value of one if $h(\mathbf{x})<0$, and zero otherwise.

Class 2: Observation providing equality information (e.g., slope failure that corresponds to a factor of safety equal to one). The corresponding likelihood function is defined as $L(\mathbf{x}) = f_{Y|\mathbf{X}}(y \mid \mathbf{x})$, where $f_{Y|\mathbf{X}}(y \mid \mathbf{x})$ is the pdf of the observation given $\mathbf{X}=\mathbf{x}$. Usually, $f_{Y|\mathbf{X}}(y \mid \mathbf{x})$ is specified by f_ε and it is defined as a Gaussian distribution (e.g., Straub & Papaioannou, 2014). Implementations of the equality information can be simplified by transforming it into an inequality information. Additional information on the transformation of equality into an inequality information and the proof can be found in Straub (2011).

The posterior distribution, $f''(\mathbf{x})$, is commonly evaluated numerically with the implementation of sampling methods as for example the Markov Chain Monte Carlo (MCMC) or the Bayesian Updating with Structural reliability methods (BUS) methods (Straub & Papaioannou, 2014). However, given that the goal of this study is the evaluation of slope safety, the focus is on the effects updating on the slope reliability. The reliability of a slope is commonly evaluated based on its complement, the failure probability, P_F. P_F is calculated as an integral under the joint pdf in the region of the outcome space $\Omega_{\mathbf{x}}$ known as the failure domain, F:

$$P_F = \Pr(F) = \int_F f(\mathbf{x}) d\mathbf{x} = \int_{g(\mathbf{x}) \le 0} f(\mathbf{x}) d\mathbf{x} = \int_{\Omega\mathbf{x}} I[g(\mathbf{x}) \le 0] f(\mathbf{x}) d\mathbf{x} \tag{2}$$

where I is the indicator function, $g(\mathbf{x})$ is the performance function that has a positive value if the state of a slope is safe for a given realization of random parameters, $g(\mathbf{x})>0$, and a non-positive value if the state of a slope is unsafe, $g(\mathbf{x}) \le 0$. Consequently, the failure domain is commonly expressed by the performance function as $F = \{\mathbf{x} \in \Omega_{\mathbf{x}} : g(\mathbf{x}) \le 0\}$.

The information on an observed slope performance can be employed to update P_F, from the initial value based on $f'(\mathbf{x})$, P'_F, to the updated one calculated with $f''(\mathbf{x})$, P''_F. This procedure corresponds to an indirect reliability updating (e.g., Schweckendiek et al., 2014). This paper implements an alternative and simpler approach, known as the direct reliability updating (Schweckendiek et al., 2014). Given that both the inequality and equality information can be expressed as inequality information, the direct reliability updating utilizes the following relation:

$$P''_F = P(F \mid Z) = \frac{P(F \cap Z)}{P(Z)} = \frac{P(\{g(\mathbf{X}) \le 0\} \cap \{h(\mathbf{X}) < 0\})}{P(h(\mathbf{X}) < 0)} \tag{3}$$

where $P''_F=P(F|Z)$ is the posterior failure probability given an event Z. The advantage of the direct reliability updating is that an estimate of P''_F can be evaluated on a set of samples from the prior distribution, $\mathbf{x}_k \sim f'(\mathbf{x})$, k=1,...,N as follows:

$$\hat{P}''_F = \hat{P}(F \mid Z) = \frac{\sum_{k=1}^{N} I[g(\mathbf{x}_k) \le 0] \cdot I[h(\mathbf{x}_k) < 0]}{\sum_{k=1}^{N} I[h(\mathbf{x}_k) < 0]} \tag{4}$$

Although the effects of updating on **x** are not explicitly examined with the direct reliability updating method, $f''(\mathbf{x})$ can be examined empirically based on the subset of samples that satisfies the constraint specified by the observation, $h(\mathbf{x})<0$.

Reliability updating of an undrained slope

Motivated by some recent slope failures in Norway (e.g., Nilsen, 2010) this study examines reliability updating of an undrained and drained slopes. Consider a slope of height H=25 m that extends for depth D=3 m until bedrock, as illustrated in Fig. 1. The slope closes an angle of β=14° with the horizontal. The height of the water table above the toe, H_w, is a random variable with a beta prior distribution, specified by the mean of 9 m and standard deviation of 0.8 m on the interval [0, H]. An undrained loading condition is examined on the slope to correspond to a situation where a sudden increase in stresses on a soil element within the slope soil mass leads to an increase in pore pressures within the element, while the volume of the element as a whole remains unchanged. Undrained conditions in slope stability analyses occur in low permeable soils (e.g., clay) and after a rapid load application.

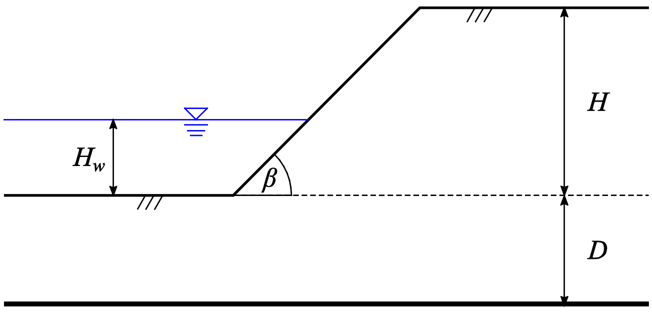

Fig. 1 Undrained slope stability analysis.

The slope stability analyses in this paper are conducted with the Janbu's direct method (Janbu, 1954) that evaluates the factor of safety of a slope based on the stability number, N_o, that is obtained from the Janbu's stability charts (e.g., Janbu, 1954):

$$F_S = \frac{N_o s_u \mu_w}{\gamma H - \gamma_w H_w} \tag{5}$$

where s_u is the undrained shear strength, specified with a normal prior distribution with the mean of 40 kPa and standard deviation of 5 kPa, γ=19 kN/m^3 is the saturated soil unit weight, γ_w=10 kN/m^3 is the water unit weight, μ_w is the submergence factor, extracted from the stability charts (Janbu, 1954). Uncertainties in the Janbu's method are modeled by an additive model error, ε, which is defined as a normal random variable with the mean of 0.01 and standard deviation of 0.049 (Duncan & Wright, 1980).

The set of random variables is defined with mutually independent components as $\mathbf{X}=[s_u, H_w, \varepsilon]^T$. The uncertainties associated with s_u and H_w are subjected to updating as they are considered as reducible, while the uncertainties in ε are considered as unreducible. Based on the prior information, an estimate of the failure probability is calculated to be $\hat{P}'_F = 0.2847$ with the Monte Carlo method on $5 \cdot 10^5$ samples. The following two cases of reliability updating are considered:

Case 1: Bayesian updating based on the observation of a stable slope with an uncertain measurement of H_w. It is assumed that a measurement of H_w=10 m is made with zero-mean normally distributed measurement error, specified by the standard deviation of 0.2 m. This observation corresponds to an inequality information, as a stable slope implies the value of F_S>1. The updated failure probability is evaluated according to Eq. 4 to be $\hat{P}''_F = 0.0657$. The observation of a stable slope under these conditions increases the reliability of the slope and favors higher values of s_u as observed from the posterior distributions in Fig. 2 and empirically estimated mean and standard deviations in Table 1.

Case 2: Bayesian updating based on the observation of a failed slope. This observation corresponds to an equality information as a failed slope implies the value of F_S=1. The equality information is transformed into an inequality formulation. Given that the slope failed (i.e., P''_F=1), the observation of a slope failure provides a valuable information on the most likely combinations of slope parameters that led to the failure. The effect of the observed slope performance on the posterior distribution of random parameters can be examined in Fig. 2 and Table 1. Given that the updating does not have a significant effect on the distribution of H_w, it can be concluded that the failure of the slope is likely to be associated with relatively low s_u values.

Table 1. Empirically estimated means and standard deviations for the undrained slope stability updating problem.

Variable	Prior		Case 1		Case 2	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
s_u	40.0	5.0	41.90	3.869	37.636	2.353
H_w	9.0	0.8	10.0	0.2	8.945	0.791
F_S	1.082	0.144	1.140	0.107	1.01	0.049

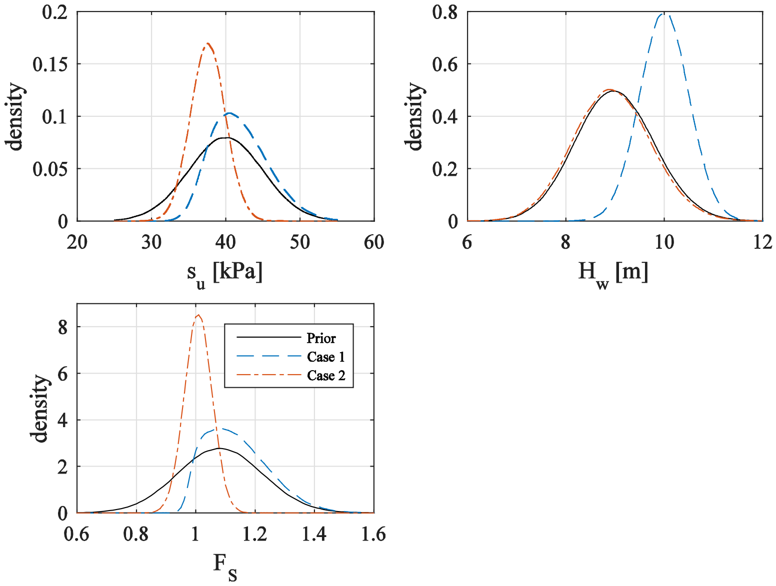

Fig. 2 Prior and posterior distributions for different updating cases of the undrained slope problem.

Reliability updating of a drained slope

Drained loading conditions occur in slopes after sufficient time has passed from a load application such that the generated excess pore pressures are dissipated. The analysis of slope stability in drained conditions is conducted with the Janbu's direct method (Janbu, 1954). The geometry of the studied drained slope is illustrated in Fig. 3 with H=17 m and β=20°. When compared to the undrained slope stability analysis in Fig. 1, the drained analysis features a groundwater table at height H'_w above the toe instead of a general water table. H'_w is specified with a prior beta distribution with the mean of 13 m and standard deviation of 1m.

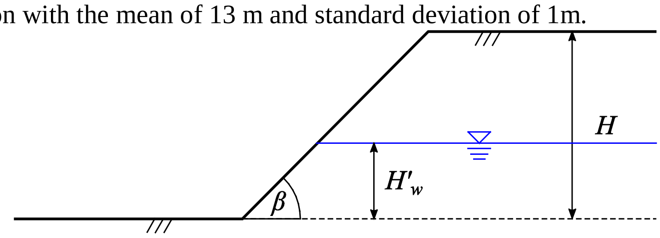

Fig. 3 Drained slope stability analysis.

The direct Janbu's method (Janbu, 1954) evaluates the factor of safety of a slope for drained conditions as follows:

$$F_S = \frac{N_{cf}(\lambda_{c\phi})c}{\gamma H}; \quad \lambda_{c\phi} = \frac{\gamma H - \gamma_w H'_w}{\mu'_w c} \tan\phi \tag{6}$$

where N_{cf} is the stability number as a function of $\lambda_{c\phi}$, μ'_w is the water seepage factor. Both N_{cf} and μ'_w are extracted from the charts associated with the direct Janbu's method (Janbu, 1954). c is cohesion, specified by a lognormal prior distribution with the mean of 10 kPa and standard deviation of 4 kPa, ϕ is the friction angle, distributed according to a normal prior with the mean of 20° and standard deviation of 5°, γ=19 kN/m^3 is the saturated soil unit weight. Uncertainties in the Janbu's method are modeled by an additive model error, ε, which is defined as a normal random variable with the mean of 0.01 and standard deviation of 0.049. The set of mutually independent random variables is defined as $\mathbf{X}=[c, \phi, H'_w, \varepsilon]^T$. The uncertainties associated with c, ϕ, and H_w are subjected to updating as they are considered as reducible, while the uncertainties in ε are considered as unreducible. Based on the prior information, an estimate of the failure probability is calculated to be $\hat{P}'_F = 0.5036$ with the Monte Carlo method on $5\cdot10^5$ samples. As defined for the undrained slope stability analysis, the following two cases of Bayesian updating are considered:

Case 1: Bayesian updating of a drained slope based on the observation of a stable slope with an uncertain measurement of H'_w.

It is assumed that a measurement of H'_w=12 m is made with a zero-mean normally distributed measurement error, specified by the standard deviation of 0.2 m. The updated failure probability is evaluated according to Eq. 4 to be $\hat{P}''_F = 0.1874$. The observation of a stable slope increases the reliability of the slope and favors higher values of c and ϕ as observed from the posterior distributions in Fig. 4 and empirically estimated mean and standard deviations in Table 2.

Case 2: Bayesian updating based on the observation of a failed slope.

This observation corresponds to an equality information, as a failed slope implies the value of F_S=1. From Fig. 4 it can be observed that the failure of the slope is likely to be associated with slightly lower c values, and ϕ values centered on the mean. The updating does not have a significant effect on the distribution of H'_w.

Table 2. Empirically estimated means and standard deviations for the drained slope stability updating problem.

Variable	Prior		Case 1		Case 2	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
c	10.0	4.0	10.994	4.252	9.775	3.529
ϕ	20.0	5.0	23.019	3.641	20.166	2.879
H'_w	13.0	2.0	12.0	0.2	13.145	1.868
F_S	1.012	0.249	1.167	0.188	1.01	0.049

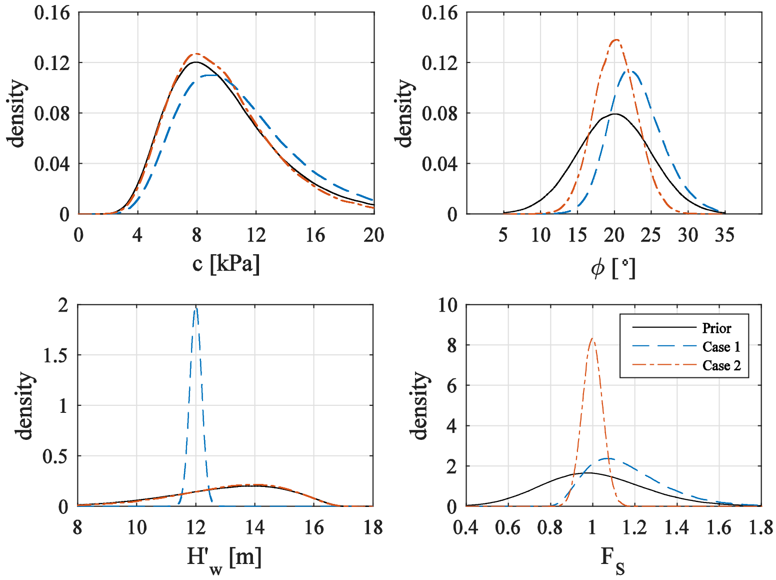

Fig. 4 Prior and posterior distributions for different updating cases of the drained slope problem.

Conclusion

The results of the conducted study demonstrated that the Bayesian framework can efficiently integrate information on observed slope performance in the safety assessment of a slope. The Bayesian framework provides a basis to explicitly update uncertainties in the safety assessment of a slope, such that they conform to the observed slope performance. The updating process is flexible and can be adapted to different types of observed information, as demonstrated by the analysis of slope survival and failure events. The application of the Bayesian framework can be further extended to consider different sources of uncertainty in slope stability analysis, spatial variability of soil properties, upgrading and repair works, code calibration, and implications on the risk management of slopes.

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